Anomaly Matching and Symmetry-protected Criticality in 1d Quantum Many-body Systems

Chang-Tse Hsieh

Kavli Institute for the Physics and Mathematics of the Universe

& Institute for Solid States Physics,

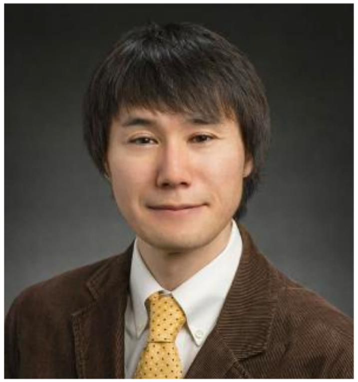
the Univ. of Tokyo

TOPMAT, IPhT, CEA, Saclay

June 13, 2018

Collaborators

Shinsei Ryu
(Chicago)



Gil Young Cho
(POSTECH)

G. Y. Cho*, **C.-T. Hsieh***, and S. Ryu, PRB 96, 195105 (2017); arXiv:1705.03892

Masaki Oshikawa
(ISSP, U of Tokyo)

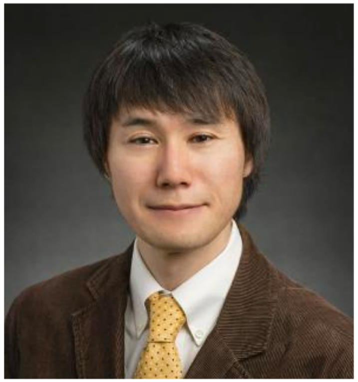
Yuan Yao
(ISSP, U of Tokyo)

Y. Yao*, **C.-T. Hsieh***, and M. Oshikawa, arXiv:1805.06885

*Equal contributions

Collaborators

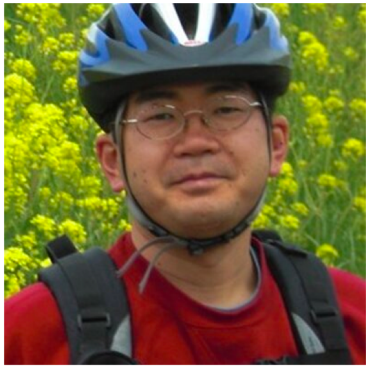
Shinsei Ryu
(Chicago)



Gil Young Cho
(POSTECH)

G. Y. Cho*, **C.-T. Hsieh***, and S. Ryu, PRB 96, 195105 (2017); arXiv:1705.03892

Masaki Oshikawa
(ISSP, U of Tokyo)



Yuan Yao
(ISSP, U of Tokyo)

Y. Yao*, **C.-T. Hsieh***, and M. Oshikawa, arXiv:1805.06885

*Equal contributions

Outline

- Introduction
- Example 1: 1d charged fermion systems
- Example 2: 1d SU(N) spin systems
- Conclusion

Outline

- Introduction
- Example 1: 1d charged fermion systems
- Example 2: 1d SU(N) spin systems
- Conclusion

Introduction

- Identifying the “phase” of a generic many-body system is an important but, in general, difficult problem

- Quite often, symmetries play an essential role in such a problem

- Various classes of phases of matter:

➢ Conventional Landau-Ginzburg-Wilson symm-breaking paradigm

➢ Topological phases: Symm-protected top. (SPT) phases

[Hasan-Kane 10; Qi-Zhang 11; Senthil 15; Chiu-Teo-Schnyder-Ryu 16; Witten 16; …]

Introduction

- Phases associated with "ingappability" (in 1d):
  - Defined regarding "whether a system with symm *can have* or *can be gapped into* a trivial – unique and symmetric – gapped ground state"

$$H_0 + \alpha H_{\text{pert}}$$

trivial ⇔ gappable   nontrivial ⇔ ingappable

- Ingappability (stability) of edge states of 2d SPT phases has been well studied, e.g. *helical edge states* of 2d QSHE

$$H_{\text{edge}} = \int dx \Psi^\dagger(x)(-iv_F\partial_x)\sigma_z\Psi(x)$$

$$\Psi(x) = (\psi_R(x), \psi_L(x))^T$$

breaks TR symm

$$\cancel{M = \psi_R^\dagger \psi_L + \text{h.c}}$$

Introduction

- Phases associated with "ingappability" (in 1d):

➢ Defined regarding "whether a system with symm *can have* or *can be gapped into* a trivial – unique and symmetric – gapped ground state"

$$H_0 + \alpha H_{\mathrm{pert}}$$

trivial ⇔ gappable   nontrivial ⇔ ingappable

- Ingappability (stability) of edge states of 2d SPT phases has been well studied, e.g. *helical edge states* of 2d QSHE

$$H_{\mathrm{edge}} = \int dx \Psi^\dagger(x)(-iv_F\partial_x)\sigma_z\Psi(x)$$

$$\Psi(x) = (\psi_R(x), \psi_L(x))^T$$

Ingappable (stable)!

$$I_{\mathrm{fw}} = g_1 \psi_R^\dagger \psi_R \psi_L^\dagger \psi_L$$

gapless

$$I_{\mathrm{Umkl}} = g_2\ e^{-i4k_F x}\psi_R^\dagger(x)\psi_R^\dagger(x+a)$$

SSB of TR $\times\ \psi_L(x+a)\psi_L(x) + \mathrm{h.c}$

[Wu-Bernevig-Zhang 06; Xu-Moore 06]

Introduction

- Phases associated with "ingappability" (in 1d):

➢ Defined regarding "whether a system with symm *can have* or *can be gapped into* a trivial – unique and symmetric – gapped ground state"

$$H_0 + \alpha H_{\mathrm{pert}}$$

trivial ⇔ gappable nontrivial ⇔ ingappable

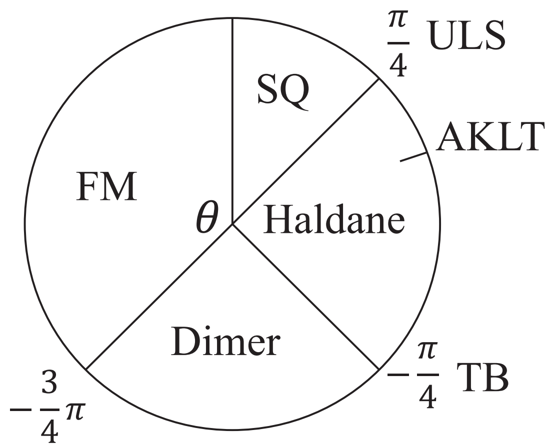
- Purely 1d (lattice) spin systems:

$$H^{s=1}_{\mathrm{HAF}} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

gapped

$$H^{s=1}_{\mathrm{BB}} = \sum_i \left[\cos\theta(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}) + \sin\theta(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1})^2\right]$$

spin-1: gappable



from Oh et al. 17

Introduction

- Phases associated with "ingappability" (in 1d):
  - Defined regarding "whether a system with symm *can have* or *can be gapped into* a trivial – unique and symmetric – gapped ground state"

$$H_0 + \alpha H_{\mathrm{pert}}$$

trivial ⇔ gappable nontrivial ⇔ ingappable

- Purely 1d (lattice) spin systems:

$$H^{s=1}_{\mathrm{HAF}} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

gapped

$$H^{s=1}_{\mathrm{BB}} = \sum_i \left[\cos\theta(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}) + \sin\theta(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1})^2\right]$$

spin-1: gappable

$$H^{s=1/2}_{\mathrm{HAF}} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

gapless

$$H^{s=1/2}_{\mathrm{MG}} = \sum_i \left(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + \frac{1}{2}\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+2} + \frac{3}{8}\right)$$

dimerized

spin-1/2: (seems) ingappable

Introduction

- Phases associated with "ingappability" (in 1d):
  - Defined regarding "whether a system with symm *can have* or *can be gapped into* a trivial – unique and symmetric – gapped ground state"

$$H_0 + \alpha H_{\mathrm{pert}}$$

trivial ⇔ gappable　　　　nontrivial ⇔ ingappable

- Purely 1d (lattice) spin systems:

$$H^{s=1}_{\mathrm{HAF}} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

gapped

$$H^{s=1}_{\mathrm{BB}} = \sum_i \left[\cos\theta(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}) + \sin\theta(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1})^2\right]$$

spin-1: gappable

$$H^{s=1/2}_{\mathrm{HAF}} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

gapless

$$H^{s=1/2}_{\mathrm{MG}} = \sum_i \left(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + \frac{1}{2}\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+2} + \frac{3}{8}\right)$$

dimerized

spin-1/2: (seems) ingappable

- Implied by the Lieb-Schultz-Mattis (LSM) theorem

Introduction

- The *LSM theorem* for 1d SU(2) spin chains:
  [Lieb-Schultz-Mattis (61); Affleck-Lieb (86); etc]

A 1d SU(2) antiferromagnetic spin chain cannot have a unique gapped GS if the spin per site is half-integral and if the lattice transl symm and SO(3) symm are strictly imposed.

- **Q1**: Given any 1d lattice model w/ both **translation** and some **on-site symm** ($G^{site} \times \mathbb{Z}^{trans}$), e.g. Hubbard or Heisenberg models, could we determine, basing on the symm and microscopic d.o.f. of the model, whether the system is ingappable?

- **Q2**: If so, could we have further constraints on the (possible) low-energy phases of this model?

➢ In this talk, we will answer these questions, focusing on the cases of $G^{site} = U(1)$ and $PSU(N)$.

Our approach

- Our approach is based on the idea of ('t Hooft) anomaly matching ['t Hooft et al. 80], which enables us to obtain some fundamental constraints on the phase diagrams.

This originates from the *non-on-site* nature of the lattice transl

[Cheng et al.15; Jian-Bi-Xu 17; Cho-Hsieh-Ryu 17; Metlitski-Thorngren 17; …]

UV

H_{lattice}

$G^{\text{site}} \times \mathbb{Z}^{\text{trans}}$

we identify a top. index, the *LSM index*, for any lattice model to characterize its phase

There is a potential "mixed anomaly" btw G and $\mathbb{Z}$.

I.e., gauging one symm causes the *breakdown* of the other.

IR

H_{EFT}

$G \times \mathbb{Z}$

(internal symm)

By "matching" the mixed anomaly at low energy,

Such an anomaly can diagnose the *ingappability* of the system! [Hsieh et al. 14]

Our approach (cont.)

- The mixed anomaly – and thus the LSM index – is a top. quantity indep. of inter-particle interactions (at either UV or IR).

➢ Because anomaly is ”preserved” under RG (‘t Hooft anomaly matching condition).

- At the lattice scale, the LSM index only depends on a quantity associated with G^{site} of the d.o.f. within a unit cell.

- Let’s see the examples $G^{site} = U(1)$ and $PSU(N)$ in the following discussion.

Outline

- Introduction
- Example 1: 1d charged fermion systems
- Example 2: 1d SU(N) spin systems
- Conclusion

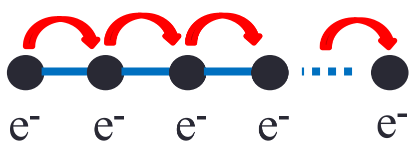
- The *LSM theorem* for 1d electron systems:
  [Oshikawa-Yamanaka-Affleck (96, 97); Oshikawa (00); etc]

A 1d electron-lattice system cannot be a trivial insulator if the filling per unit cell is fractional and if the lattice transl symm and charge conservation are strictly imposed.

1d electron system with translation symmetry

- A simple model: tight-binding model of 1d spinless fermions

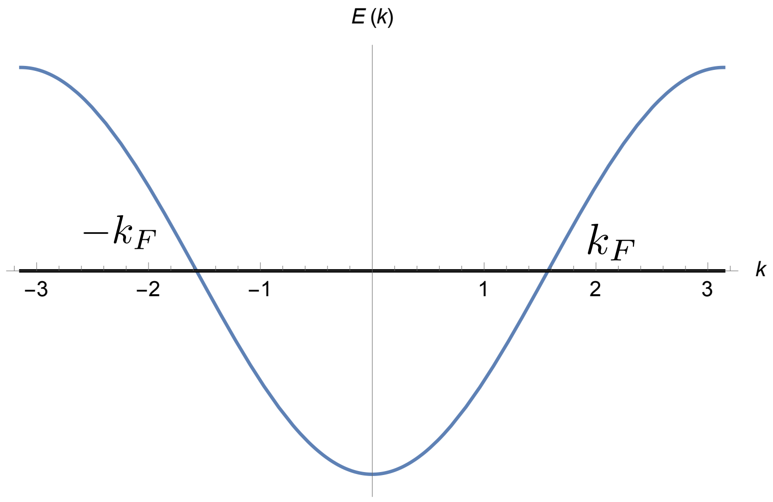
$$H = -t\sum_{x}^{L}(c_x^\dagger c_{x+1} + h.c.) - \mu \sum_{x}^{L} c_x^\dagger c_x$$



$$U(1)_Q: \quad c_x \to e^{i\phi} c_x,$$
$$\mathbb{Z}_{trans}: \quad c_x \to c_{x+1}.$$

$$|GS\rangle \propto \Big(\prod_{|k|\le k_F} c_k^\dagger\Big)|\mathrm{vac}\rangle$$

$$\text{filling } \nu = \frac{N_e}{L} = \frac{k_F}{\pi}$$



1d electron system with translation symmetry

- The continuum IR limit of the theory:

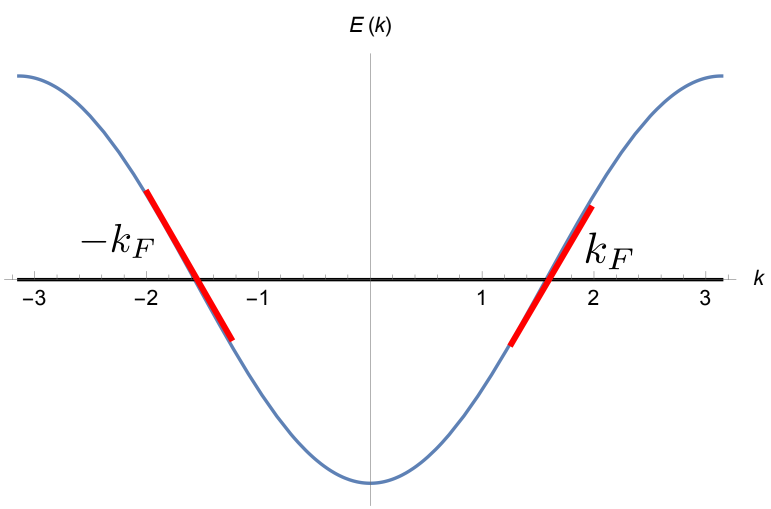
$$c_x \approx \psi_R(x)e^{ik_F x} + \psi_L(x)e^{-ik_F x}$$

$$H = \int dx\ \Psi^\dagger(x)(-iv_F\partial_x)\sigma_z\Psi(x) \qquad \Psi(x) = (\psi_R(x), \psi_L(x))^T$$

Symmetry in the low-energy theory:

$$U(1)_Q : \quad \Psi(x) \to e^{i\phi}\Psi(x),$$
$$\mathbb{Z}_{trans} : \quad \Psi(x) \to e^{ik_F\sigma^z}\Psi(x)$$

- ➢ Taking ν = 1/2, we have a local and internal symm $U(1)_Q \times Z_2$.
- ➢ For generic ν, we have $U(1)_Q \times Z$



Anomalies in the low-energy theory

- There is a "discrete" **axial/chiral/mixed anomaly** in the Dirac theory

$$H = \int dx\ \Psi^\dagger(x)(-iv_F\partial_x)\sigma_z\Psi(x)$$

$$U(1) : \Psi \to e^{i\phi}\Psi \qquad \text{vector}$$

$$\mathbb{Z} : \Psi \to e^{ik_F\sigma_z}\Psi = e^{i\pi\nu\sigma_z}\Psi \qquad \text{axial}$$

- That is, the part. func. in the presence of a $U(1)$ field is in general not invariant under the axial transf [Cho-Hsieh-Ryu 17]

$$Z(A_{U(1)}) \xrightarrow{\text{axial}} e^{2\pi i\nu\times\text{integer}} Z(A_{U(1)})$$

=> axial anomaly is characterized by ν, the filling per unit cell!

Implication of the mixed anomaly

- **IR**: If $\nu \neq$ integer, the low-energy theory is *anomalous*; it must be either gapless or, when perturbed by (symmetric) interactions, gapped with spontaneous symm breaking.

- **UV** (lattice): If the filling per unit cell is not integral, the system does not allow a unique gapped ground state; it must be in a gapless phase or a gapped phase breaking the transl symm.

  - This is nothing but the LSM theorem for an electron system!

Implication of the mixed anomaly

- For example, a half-filled spinless ferm ($\nu = ½$) is ingappable

$$\text{e.g.} \quad -t\sum_x (c_x^\dagger c_{x+1} + h.c.) + U\sum_x n_x n_{x+1}$$

(U/t >>1, the system is gapped w/ **SSB** of transl)

while two half-filled (spinful) ferm ($\nu_{\text{tot}} = 1$) is gappable

$$\text{e.g.} \quad -t\sum_x (c_{\uparrow,x}^\dagger c_{\uparrow,x+1} + c_{\downarrow,x}^\dagger c_{\downarrow,x+1}) + V\sum_x c_{\uparrow,x}^\dagger c_{\downarrow,x} + h.c.$$

(V/t >>1, the system is **trivially** gapped)

LSM indices of 1d charged fermion systems with translational symmetry

- By anomaly matching, one can identify the LSM index of a generic 1d charged fermion system [Cho-Hsieh-Ryu 17]

$$\boxed{\text{LSM index } \mathcal{I} = (\text{total charges per unit cell}) \mod 1}$$

UV

$\mathrm{H}_{\mathrm{lattice}}$

$U(1)^{\mathrm{site}} \times \mathbb{Z}^{\mathrm{trans}}$

IR

$\mathrm{H}_{\mathrm{EFT}}$

$U(1) \times \mathbb{Z}$

(internal symm)

Any mixed anomaly is represented by an S^1 number $\in \mathbb{R}/\mathbb{Z}$

- Ingappability for more general systems (Tomonaga–Luttinger liquid) can be checked by bosonization [Cho-Hsieh-Ryu (17)]

- In summary, we have the LSM physics for 1d fermionic systems from the perspective of anomaly matching:

  A 1d electron-lattice system cannot be a trivial insulator if the filling per unit cell is fractional and if the lattice transl symm and charge conservation are strictly imposed.

- This agrees with previous works by Oshikawa-Yamanaka-Affleck (96, 97), Oshikawa (00), etc.

Outline

- Introduction
- Example 1: 1d charged fermion systems
- Example 2: 1d SU(N) spin systems
- Conclusion

1d spin chain with translation symmetry

- The *LSM theorem* for 1d $SU(2)$ spin chains:
  [Lieb-Schultz-Mattis (61); Affleck-Lieb (86); etc]

A 1d SU(2) antiferromagnetic spin chain cannot have a unique gapped GS if the spin per site is half-integral and if the lattice transl symm and SO(3) symm are strictly imposed.

1d spin chain with translation symmetry

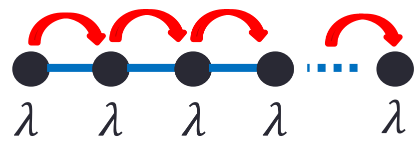
- A generalization to the case of $SU(2N)$ spin chain was also known: [Affleck-Lieb (86)]

*A 1d SU(2N) antiferromagnetic spin chain cannot have a unique gapped GS if the **Young tableau rep** per site has an odd number of boxes and if the lattice transl symm and PSU(2N) symm are strictly imposed.*

1d spin chain with translation symmetry

- How about for an $SU(N)$ chain with a generic Young-tableau (YT) rep λ per site and with both $PSU(N)$ and transl Z^{trans} symm?

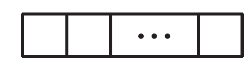
  ➢ A typical example is the (generalized) HAF model



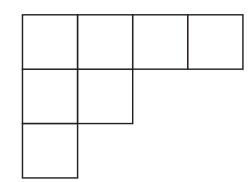
$$\mathscr{H}_{\text{HAF}} = \sum_{\langle i,j\rangle,\alpha,\beta} J S^{\alpha}_{i,\beta} S^{\beta}_{j,\alpha}, \quad J > 0$$

$$\left[S^{\alpha}_{i,\beta}, S^{\gamma}_{j,\delta}\right] = \delta_{i,j}\left(\delta^{\alpha}_{\delta} S^{\gamma}_{i,\beta} - \delta^{\gamma}_{\beta} S^{\alpha}_{i,\delta}\right)$$

$N = 2 : \lambda = \{2s\}$



$N > 2 : \lambda = \{4, 2, 1\}$
(for example)



- We will answer this question, again, by the anomaly matching argument

LSM indices of 1d *SU*(*N*) spin systems with translational symmetry

- Let's identify the LSM index for a 1d $SU(N)$ lattice model from the form of the mixed anomaly of $PSU(N) \times \mathbb{Z}$.

  [Yao-Hsieh-Oshikawa 18]

$$\boxed{\text{LSM index } \mathcal{I}_N = (\#\text{ of YT boxes per unit cell}) \mod N}$$

UV

$\mathrm{H}_{\text{lattice}}$

$PSU(N)^{\text{site}} \times \mathbb{Z}^{\text{trans}}$

IR

H_{EFT}

$PSU(N) \times \mathbb{Z}$

(internal symm)

Any mixed anomaly is represented by a mod N integer $\in \mathbb{Z}_N$

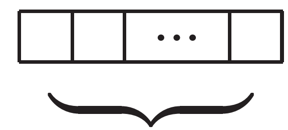
Math fact: $H^3(PSU(N) \times \mathbb{Z}, U(1))/H^3(PSU(N), U(1)) \cong \mathbb{Z}_N$

LSM/SPT: Cheng et al.15; Jian-Bi-Xu 17; Cho-Hsieh-Ryu 17; Metlitski-Thorngren 17; …

Check with simple examples:

- ***N* = 2**: For a system w/ spin *s* per unit cell we have

$$\mathcal{I}_2 = 2s \mod 2$$



$2s$ YT boxes

- ***N* = even**: For a system w/ SU(N) spin λ w/ an odd # of boxes

$$\mathcal{I}_N \neq 0 \mod N$$

➢ Agree with the LSMA theorem!

- **Q1**: Given any 1d lattice model w/ both **translation** and some **on-site symm** ($G^{site} \times \mathbb{Z}^{trans}$), e.g. Hubbard or Heisenberg models, could we determine, basing on the symm and microscopic d.o.f. of the model, whether the system is ingappable?

- **Q2**: If so, could we have further constraints on the (possible) low-energy phases of this model?

- **Q1**: Given any 1d lattice model w/ both **translation** and some **on-site symm** ($G^{site} \times \mathbb{Z}^{trans}$), e.g. Hubbard or Heisenberg models, could we determine, basing on the symm and microscopic d.o.f. of the model, whether the system is ingappable?

- **Q2**: If so, could we have further constraints on the (possible) low-energy phases of this model?

Constraints on the low-energy phases

- The value of the LSM index I_N can be used to further **constrain** the *GSD* or the *possible universality class* when the system is in a gapped or a critical phase, respectively:

➢ GSD of a gapped phase: $$\mathrm{GSD} \in \frac{N}{\gcd(\mathcal{I}_N, N)}\mathbb{N} \qquad (1)$$

➢ $SU(N)_k$ WZW CFT with transl symm $g \to e^{2\pi i m/N} g$:

$$\mathcal{I}_N = km \mod N \qquad (2)$$

=> For $N = 2$, (2) agrees with Furuya-Oshikawa 15

Anomalies in *SU*(*N*) WZW theories

- The most natural univ classes of a critical *SU*(*N*) spin model is the *SU*(*N*) WZW theories

$$kI(g) = \frac{k}{8\pi}\int_{M_2} dtdx\mathrm{Tr}\left(\partial_\mu g^{-1}\partial^\mu g\right) + k\Gamma_{\mathrm{WZ}}$$

level

symm:

vector $PSU(N) : g \to wgw^{-1},\ \ w \in SU(N)$

axial $\mathbb{Z}_n(\mathrm{trans}) : g \to e^{2\pi i m/N} g,\ \ m \in \{0, 1, ..., N-1\}$

$n = N/\gcd(m, N)$

- Mixed anomaly of the *PSU(N)* $\times\ \mathbb{Z}_n$ symm [Yao-Hsieh-Oshikawa 18]:

$$Z(A_{PSU(N)}) \xrightarrow{\text{axial}} e^{2\pi i \frac{km}{N}\times \text{integer}} Z(A_{PSU(N)})$$

characterized by *km*/*N*, or *km* mod *N*

- Our prediction agrees with known examples in previous studies of $SU(N)$ models.

	Model	YT	$\mathcal{I}_N$	GSD	IR CFT; m	Mixed anomaly
Greiter et al. 07	$SU(3)$ trimer model [43]	$\square$	$1 \bmod 3$	$3 \in 3\mathbb{N}$	-	-
	$SU(3)$ **10**-VBS model [43]	$\square\square\square$	$0 \bmod 3$	$1 \in 1\mathbb{N}$	-	-
Greiter-Rahel 07	$SU(6)$ **70**-VBS model [44]	$\begin{array}{ll}\square & \square \\ \square & \end{array}$	$3 \bmod 6$	$2 \in 2\mathbb{N}$	-	-
Takhtajan; Babujian 82	S-3/2 TB model[45, 46]	$\square\square\square$	$1 \bmod 2$	-	$SU(2)_3$ WZW; 1	$1 \bmod 2$
Andrei-Johannesson 84 Johanness 86	$\mathscr{H}^{[3,2]}$ AJ model[47, 48]	$\square\square$	$2 \bmod 3$	-	$SU(3)_2$ WZW; 1	$2 \bmod 3$
Rachel et al. 09	$SU(3)$ 1×2-YT HAF[49, 50]	$\square\square$	$2 \bmod 3$	-	$SU(3)_1$ WZW; 2	$2 \bmod 3$
Dufour et al. 15	$SU(9)$ 2×1-YT HAF[51]	$\begin{array}{l}\square \\ \square\end{array}$	$2 \bmod 9$	-	$SU(9)_1$ WZW; 2	$2 \bmod 9$
Lecheminant 15	$SU(3)$ 2-leg ladder [52]	$\square \otimes \square$	$2 \bmod 3$	-	$SU(3)_1$ WZW; 2	$2 \bmod 3$

Y. Yao, C.-T. Hsieh, and M. Oshikawa, arXiv:1805.06885

In summary, if a spin model with an exact $SU(N)$ spin-rotation and transl symm has a **nontrivial LSM index**, i.e., the total umber of Young-tableau boxes per unit cell is not divisible by N, the system must have either

- degenerate gapped ground states, with the multiplicity (1), or
- gapless excitations, which is symm protected/respected. If the low-energy EFT is given by an $SU(N)$ WZW theory, its level is constrained by (2).

Symmetry enlargement

- An $SU(N)$ model might have an $SU(N')_{k'}$ WZW critical theory w/ $N' > N$. Such a critical theory is constrained by the LSM index as

$$\mathcal{I}_N \cdot \frac{N'}{\gcd(N', k'm')} = 0 \mod N$$

- For example, an $SU(2)$ spin chain with a half-integer spin per unit cell does ***not*** admit an $SU(N')$ WZW critical theory for any odd integer N'

➢ This explains $SU(3)$ symm are only found in integer-spin models [Uimin 70; Lai 74; Sutherland 75; Chen *et al.* 15]

Critical spin-1 chain: SU(3) symm is enhanced in the lattice model

Critical spin-2 chain: SU(3) symm emerges only in the thermodynamic limit

Symmetry enlargement

- More on the Uimin-Lai-Sutherland model:

$$H_{\mathrm{ULM}} = \underbrace{\sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + (\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1})^2}_{\text{SU(2) spin-1}} \propto \underbrace{\sum_i \sum_{A=1}^{8} T_i^A T_{i+1}^A}_{\text{SU(3) “spin” in the fund rep}}$$

SU(2) spin-1 ⇩ original index: $I_2 = 0 \mod 2$

SU(3) “spin” in the fund rep ⇩ a “finer” index: $I_3 = 1 \mod 3$

➢ Both indices are consistent with the existence of the $SU(3)_1$ WZW critical theory!

Outline

- Introduction
- Example 1: 1d charged fermion systems
- Example 2: 1d SU(N) spin systems
- Conclusion

Conclusion

- We apply the idea of ('t Hooft) anomaly matching to study 1d condensed matter systems – many-body systems in general – in the presence of both lattice transl and some on-site symm.

This originates from the *non-on-site* nature of the lattice transl

[Cheng et al.15; Jian-Bi-Xu 17; Cho-Hsieh-Ryu 17; Metlitski-Thorngren 17; …]

UV

$\mathrm{H}_{\mathrm{lattice}}$

$G^{\mathrm{site}} \times \mathbb{Z}^{\mathrm{trans}}$

IR

$\mathrm{H}_{\mathrm{EFT}}$

$G \times \mathbb{Z}$

(internal symm)

There is a potential "mixed anomaly" btw G and $\mathbb{Z}$.

I.e., gauging one symm causes the *breakdown* of the other.

we identify a top. index, the *LSM index*, for any lattice model to characterize its phase

By "matching" the mixed anomaly at low energy,

Such an anomaly can diagnose the *ingappability* of the system! [Hsieh et al. 14]

Thank You!