

TOPMAT 2018

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MPI - PKS Dresden

Part I

**New phase transitions of Composite
Fermions**

Part II

**Bosonization and shear sound in 2D
Fermi liquids**

New phase transitions of Composite Fermions



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- Phase transition between Pfaffian and composite fermi liquid in bilayer graphene by turning magnetic field.
- Stoner transition of composite fermi liquids in AlAs (arXiv: 1802.02167).

Zoo of fractionalized liquids

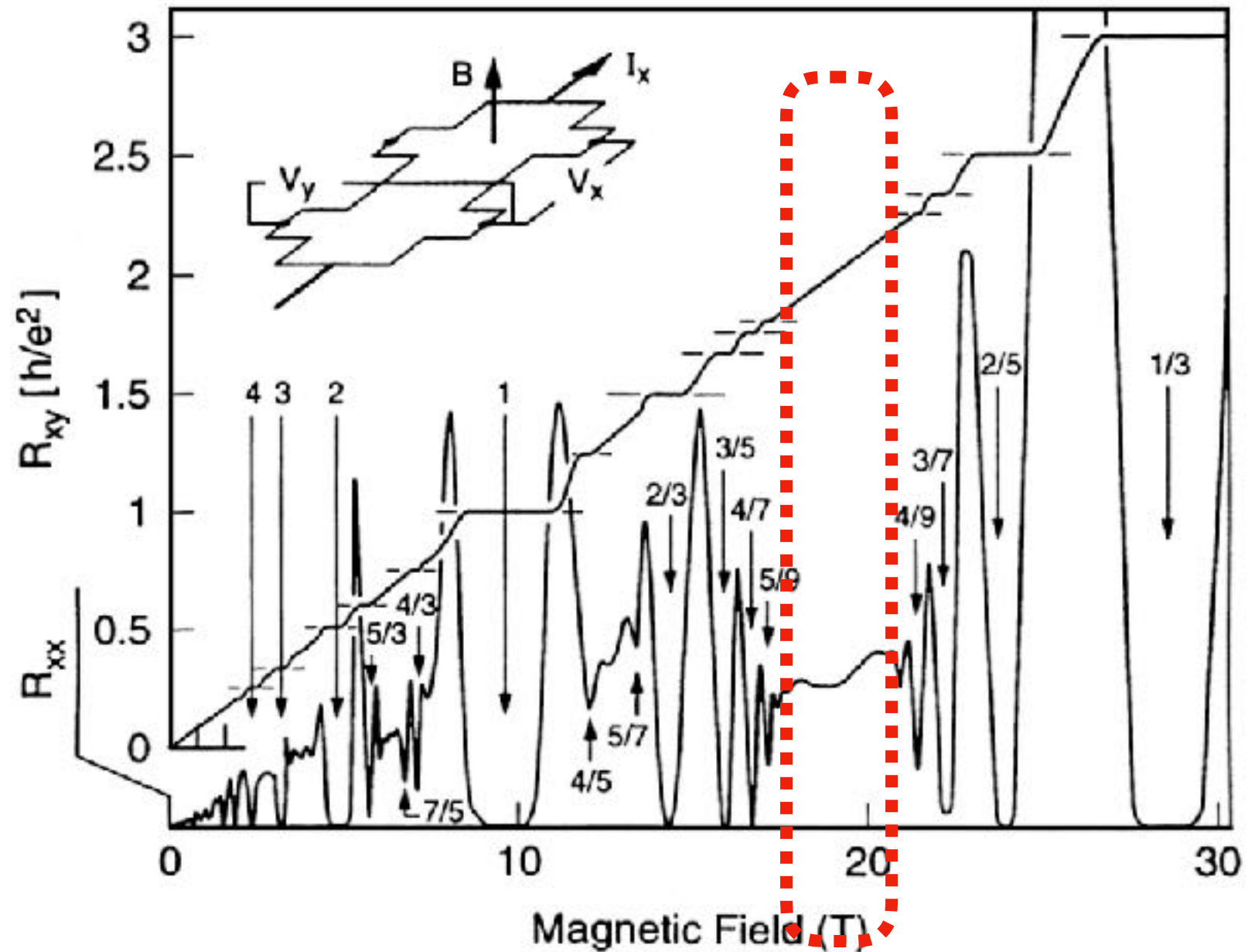
A gift from nature:

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

$$\sigma_{xx} = 0$$

Any phase of matter with
fractional ν must have
fractionalized
excitations

Gapless phases don't have
quantised σ_{xy}



We believe this is a gapless
fractionalized phase of matter:
Composite Fermi Liquid

$$\nu = \frac{1}{2}$$

The composite Fermi liquid

Consider spin-full electrons at

$$\nu = \frac{1}{2}$$

Slave-boson

Spinfull
neutral fermion

Electron $\longrightarrow$

$$c_{\sigma r}^{\dagger} = f_{\sigma r}^{\dagger} b_r^{\dagger}$$

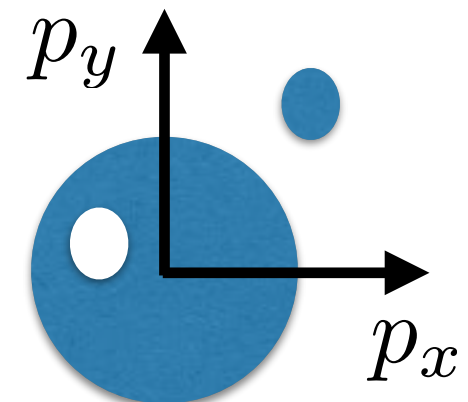
Spinless
charged boson $\longleftarrow$

Boson forms Laughlin state:

$$\nu_b = \nu_e = \frac{1}{2}$$

$$\Psi_b = \prod_{i < j} (z_i - z_j)^2 e^{-\frac{|z_i|^2}{4l^2}}$$

Fermion forms fermi sea:



**composite fermion fermi
surface**

The composite Fermi liquid

Consider spin-full electrons at

$$\nu = \frac{1}{2}$$

Slave-boson

Spinfull
neutral fermion
↓

Electron $\longrightarrow c_{\sigma r}^{\dagger} = f_{\sigma r}^{\dagger} b_r^{\dagger} \longleftarrow$ Spinless
charged boson

$$\nu_b = \nu_e = \frac{1}{2}$$

State of b	Mott - CDW	Laughlin state
Phase of electrons	U(1) spinon Fermi surface	Composite Fermi liquid
Name of f	Spinon	Composite fermion

Wave function of Composite Fermi Liquid

The key to quantum Hall energetics

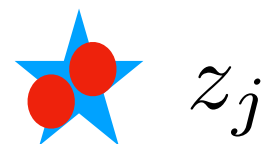
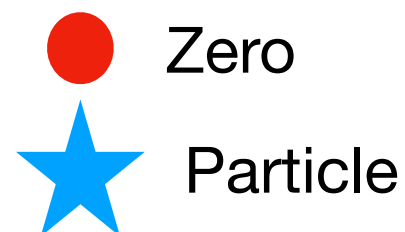
How to make particles stay as far away from each other as possible within a Landau level at filling ν :

$$\Psi = \prod_{i < j} (z_i - z_j)^{\frac{1}{\nu}} \quad z = x + iy$$

We get the bosonic Laughlin state at

$$\nu = \frac{1}{2}$$

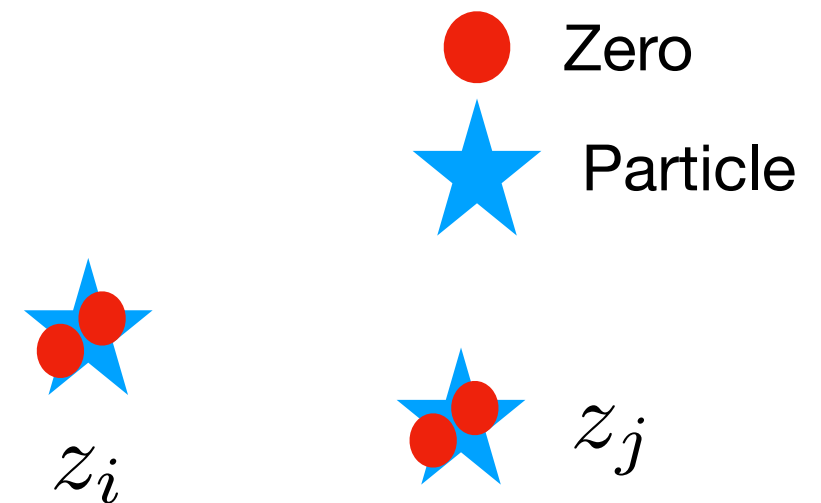
$$\Phi_{bose} = \prod_{i < j} (z_i - z_j)^2 e^{-\frac{|z_i|^2}{4l^2}}$$



Wave function of Composite Fermi Liquid

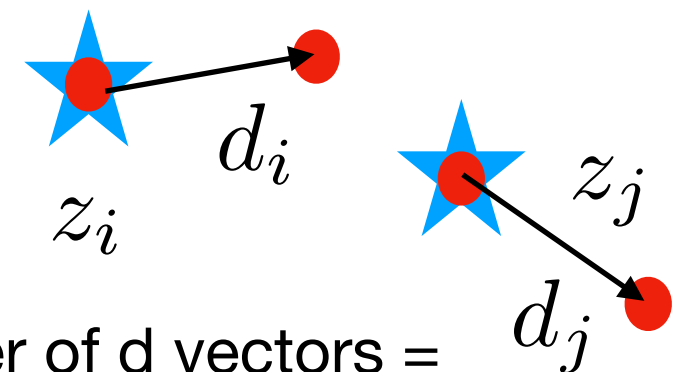
We get the bosonic Laughlin state at $\nu = \frac{1}{2}$

$$\Phi_{bose} = \prod_{i < j} (z_i - z_j)^2 e^{-\frac{|z_i|^2}{4l^2}}$$

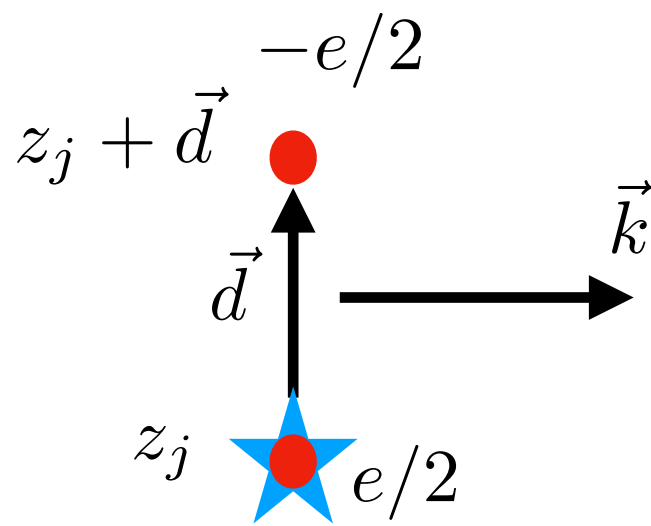


To get fermions: displace the zeroes as little as possible to be able to anti-symmetrize

$$\Psi_{fermi} = \mathcal{A} \left(\prod_{i < j} (z_i - z_j)(z_i - z_j - d_i - d_j) e^{-\frac{|z_i|^2}{4l^2}} \right)$$



Number of d vectors =
Number of particles

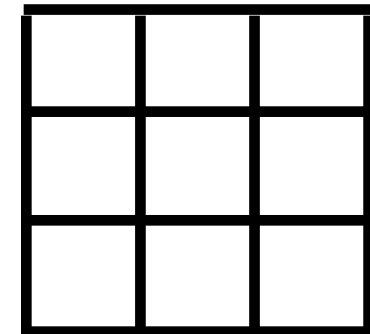


$$\mathbf{d}_i = -l^2 \hat{z} \times \mathbf{k}_i$$

Composite fermions on Torus

Single particle translations form a discrete lattice on a finite size torus

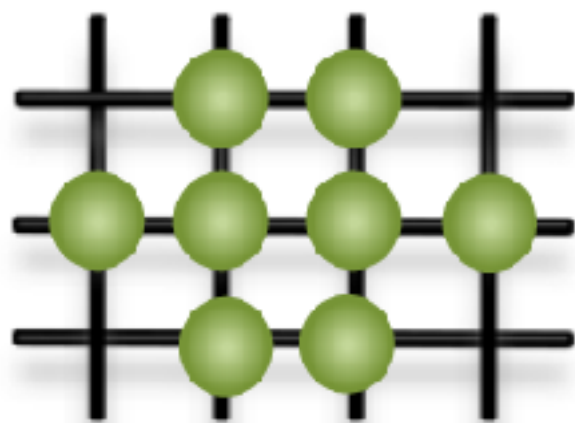
$$\mathbf{d} \in \frac{m_1 \mathbf{L}_1 + m_2 \mathbf{L}_2}{N_\phi}, m_{1,2} \in \mathbb{Z} \bmod(N_\phi)$$



$\hat{t}_i(\mathbf{d})$ Translation of particle i by vector $\mathbf{d}$

For any given set of N distinct $\{\mathbf{d}_i\}$

We can construct a Composite Fermion trial wave-function



$$|\Psi_{\text{CFL}}(\{\mathbf{d}_i\})\rangle = \det(\hat{t}_j(\mathbf{d}_i)) |\Phi_{1/2}^{\text{Bose}}\rangle$$

$$\mathbf{d}_i = -l^2 \hat{z} \times \mathbf{k}_i$$

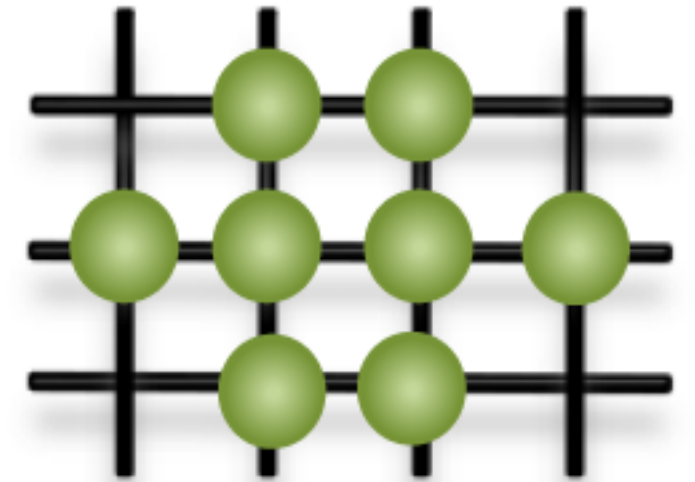
E. H. Rezayi and F. D. M. Haldane, Phys. Rev. Lett. **84**, 4685 (2000).

Shao, Kim, Haldane, & Rezayi, PRL (2015).

Composite fermions on Torus

$$|\Psi_{\text{CFL}}(\{\mathbf{d}_i\})\rangle = \det(\hat{t}_j(\mathbf{d}_i)) |\Phi_{1/2}^{\text{Bose}}\rangle, \quad \mathbf{d}_i \equiv -l^2 \hat{\mathbf{z}} \times \mathbf{k}_i$$

$$\mathbf{d} \in \frac{m_1 \mathbf{L}_1 + m_2 \mathbf{L}_2}{N_\phi}, \quad m_{1,2} \in \mathbb{Z} \bmod(N_\phi)$$



Given a set of occupied states we can predict many body momenta

$$\mathbf{K} = \frac{\hat{\mathbf{z}}}{l^2} \times \sum_i \mathbf{d}_i = 2\pi \left(-\frac{\sum_i m_{2i}}{L_1}, \frac{\sum_i m_{1i}}{L_2} \right)$$

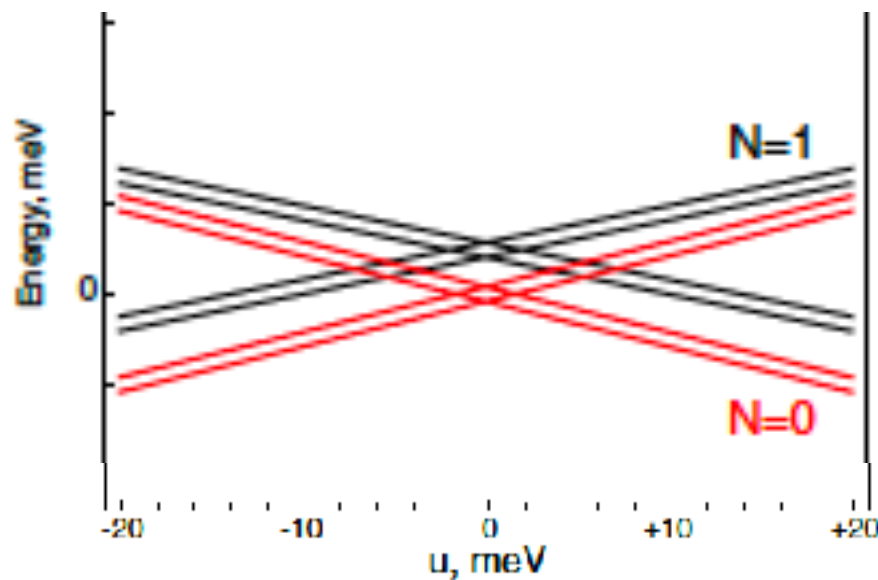
Composite fermion kinetic energy

$$E[\{\mathbf{k}_i\}] = E_0 + \frac{1}{N_e} \sum_{i < j} \epsilon(\mathbf{k}_i - \mathbf{k}_j)$$

$$\approx E_0 + \frac{1}{N_e} \sum_{i < j} \frac{|\mathbf{k}_i - \mathbf{k}_j|^2}{2m^*},$$

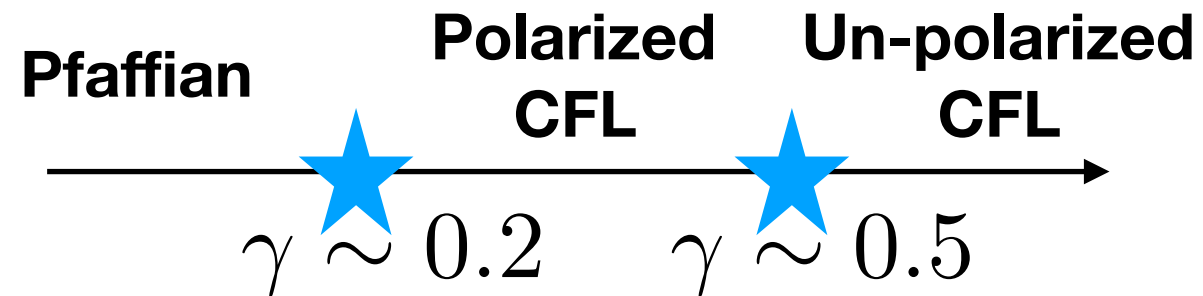
Shao, Kim, Haldane, & Rezayi, PRL (2015).

A smoother transition between the Pfaffian and the CFL

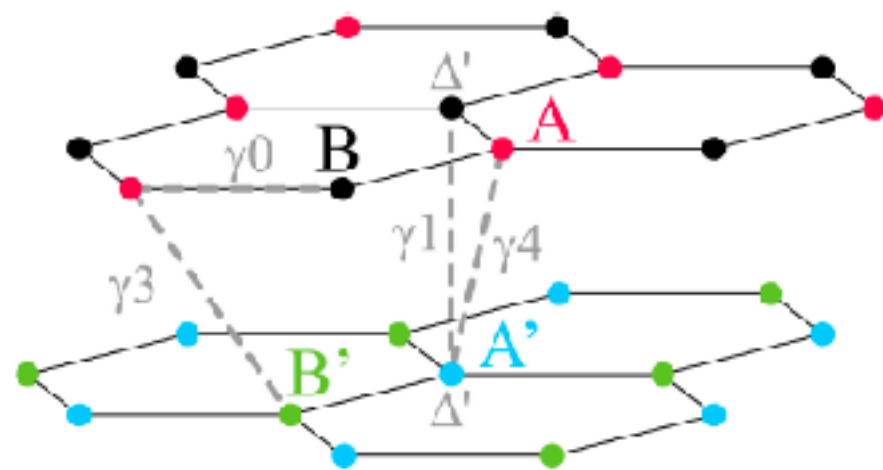


Imagine 2 components continuously rotating from N=0 LL into a N=1LL

$$|\gamma\rangle = \gamma|0, \uparrow\rangle + (1 - \gamma)|1, \downarrow\rangle$$

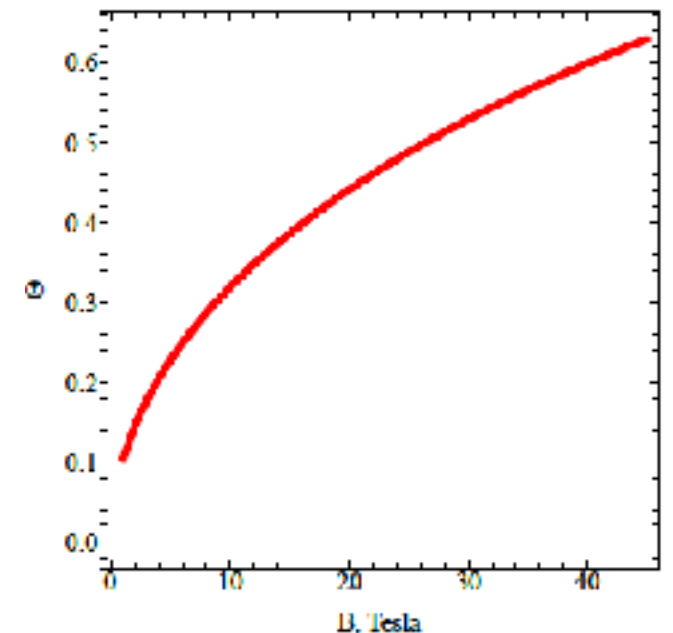


In the N=1 LL of bilayer graphene one can achieve version of this:



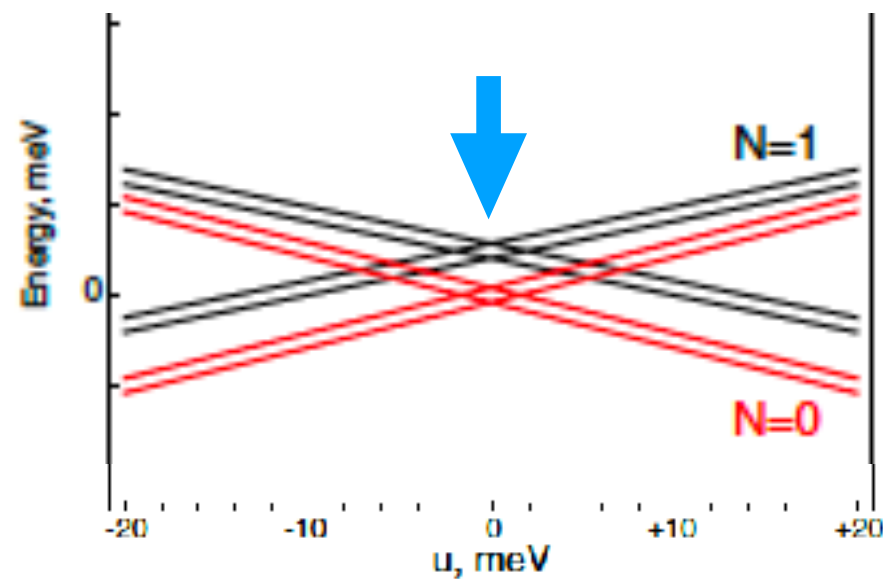
$$\psi_{1+} = \begin{pmatrix} \cos \Theta |1\rangle \\ 0 \\ \cos \Phi \sin \Theta |0\rangle \\ \sin \Phi \sin \Theta |0\rangle \end{pmatrix} \quad \begin{matrix} A \\ B' \\ A' \\ B \end{matrix}$$

$$\psi_{1-} = \begin{pmatrix} 0 \\ \cos \Theta |1\rangle \\ \sin \Phi \sin \Theta |0\rangle \\ \cos \Phi \sin \Theta |0\rangle \end{pmatrix}$$



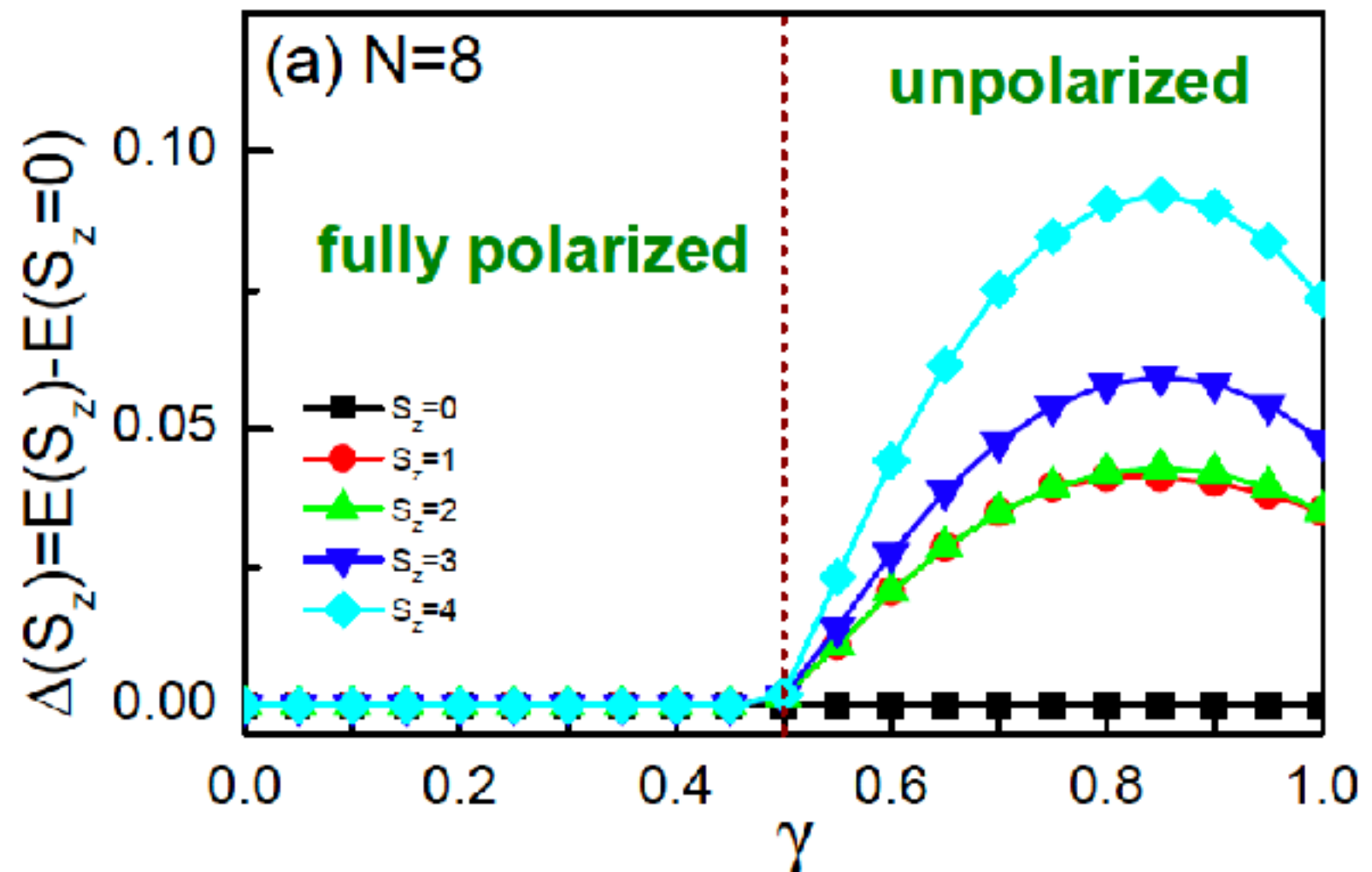
SU(2) magnet to SU(2) singlet

Consider SU(2) 2-component system:



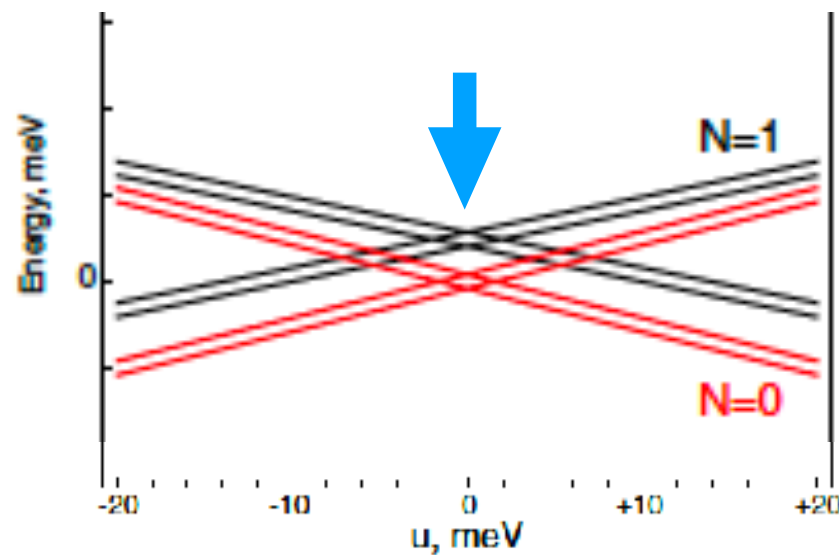
$$|\gamma\rangle = \gamma|0, \uparrow\rangle + (1 - \gamma)|1, \downarrow\rangle$$

ED Spectrum
Torus



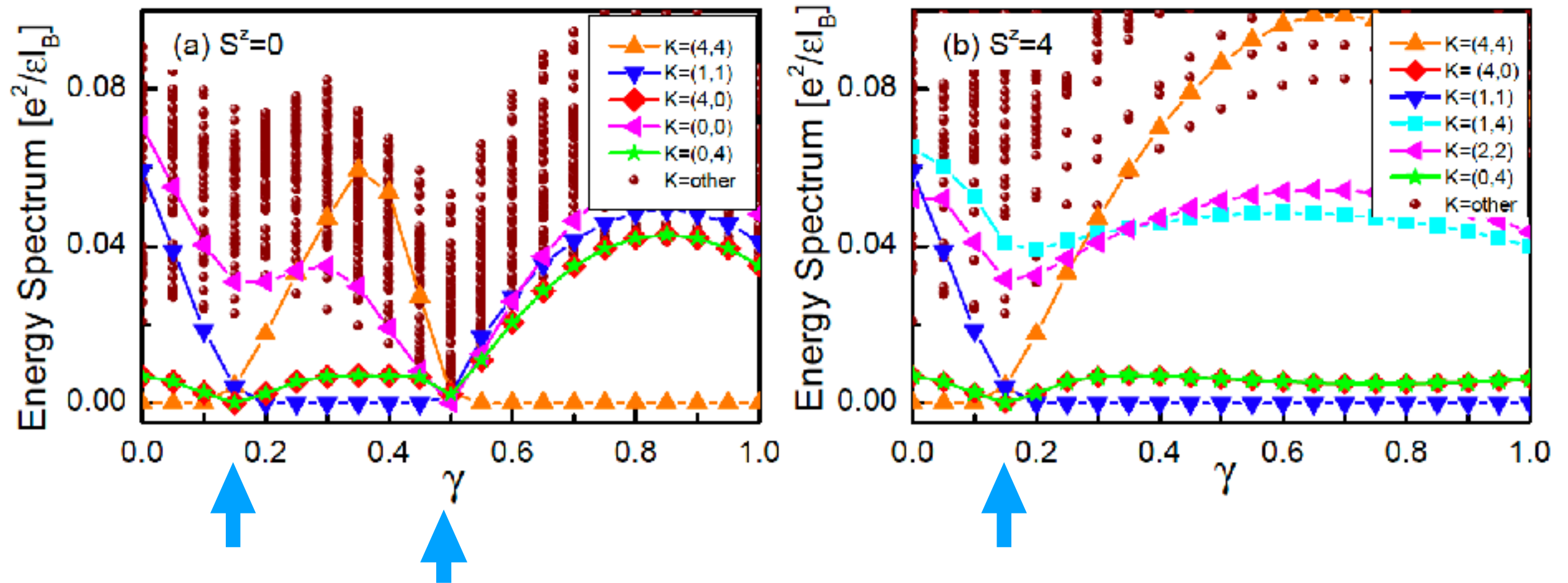
Phase transition of polarized states

Consider SU(2) 2-component system:

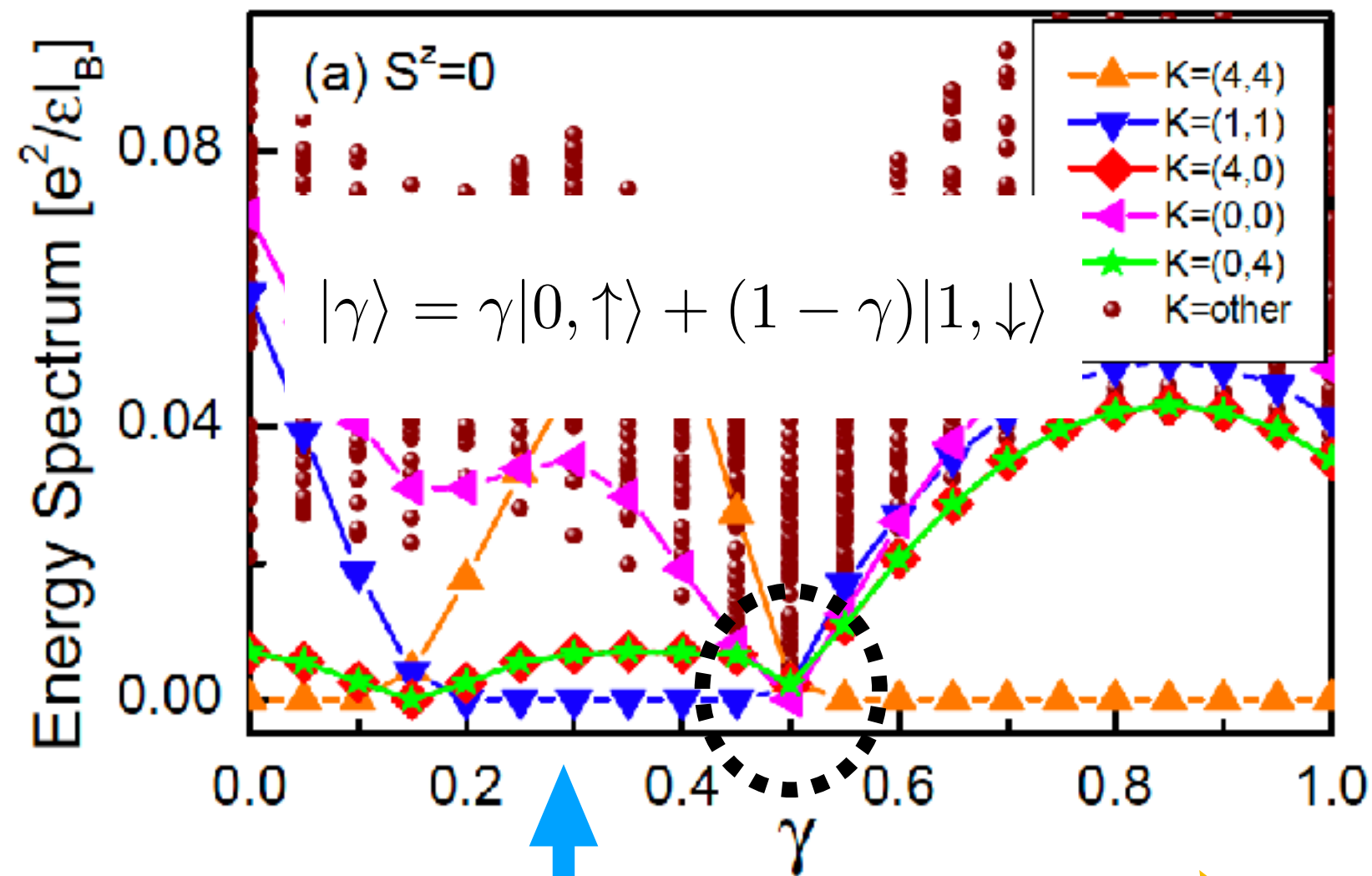


$$|\gamma\rangle = \gamma|0, \uparrow\rangle + (1 - \gamma)|1, \downarrow\rangle$$

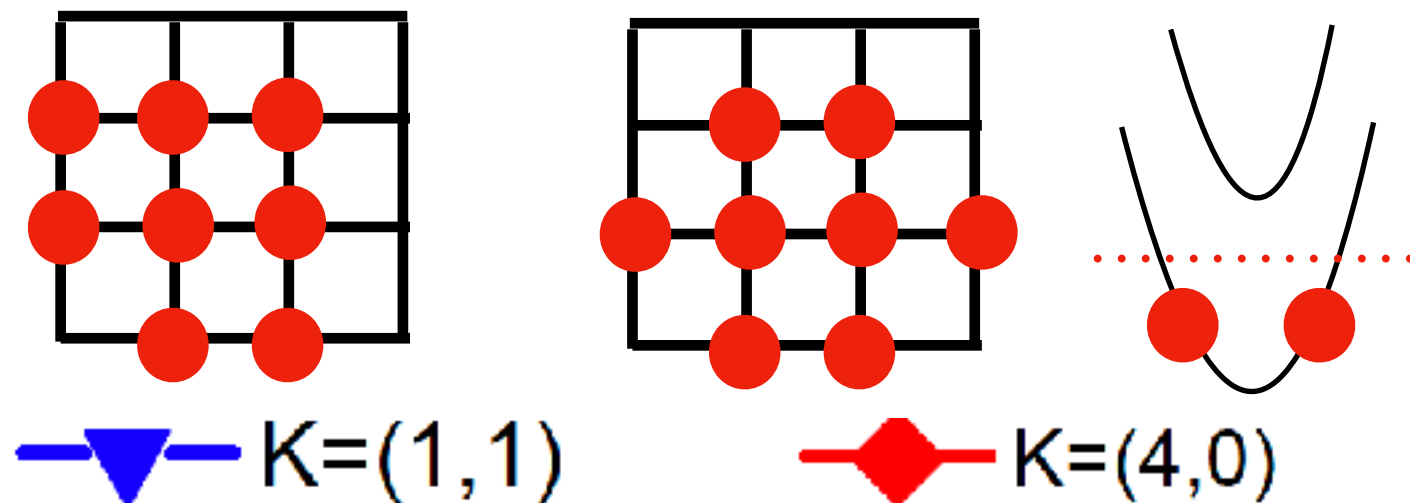
ED Spectrum Torus



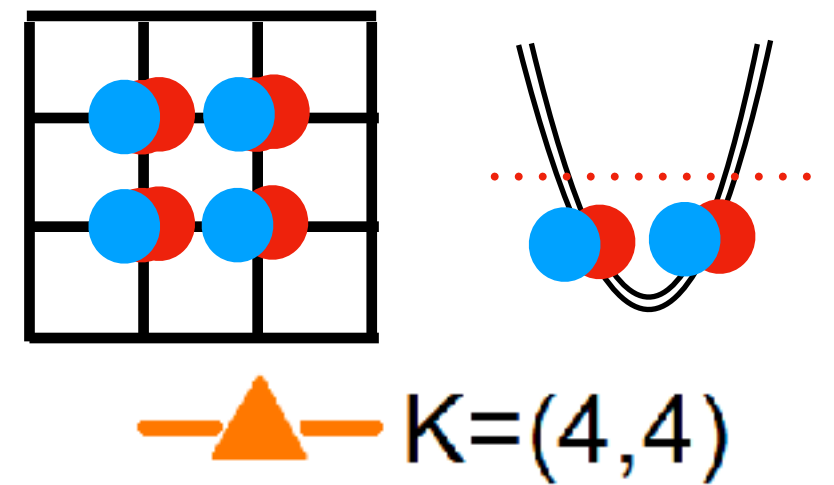
Stoner transition of CFL states



$$S^2 = S_z^{max} (S_z^{max} + 1)$$



$$S^2 = 0$$

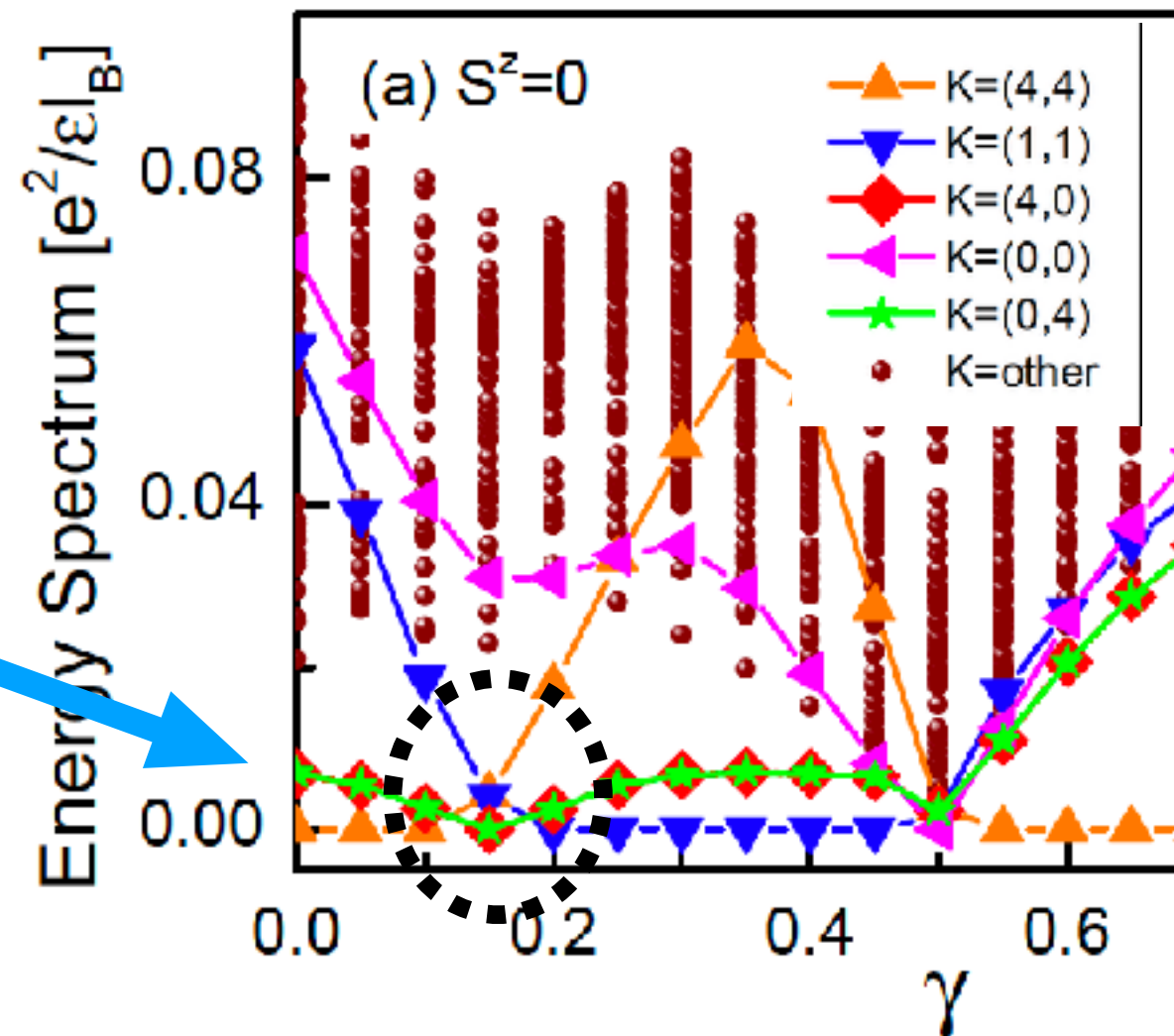


Pfaffian to CFL transition

$$|\gamma\rangle = \gamma|0, \uparrow\rangle + (1 - \gamma)|1, \downarrow\rangle$$

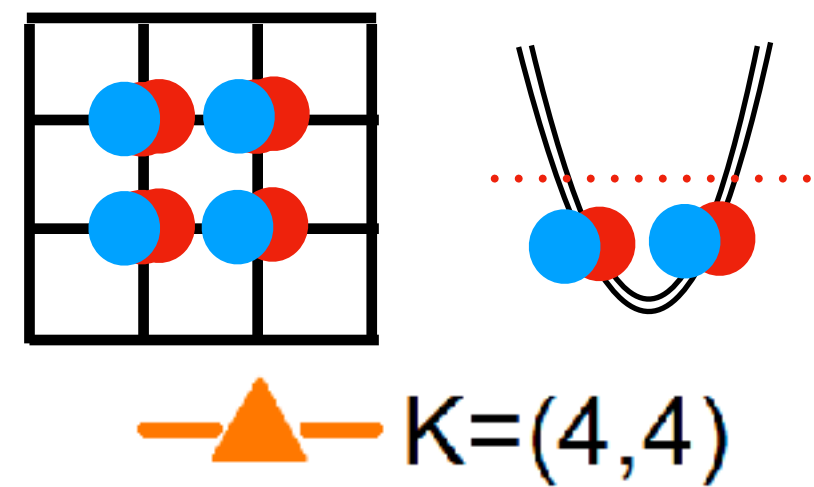
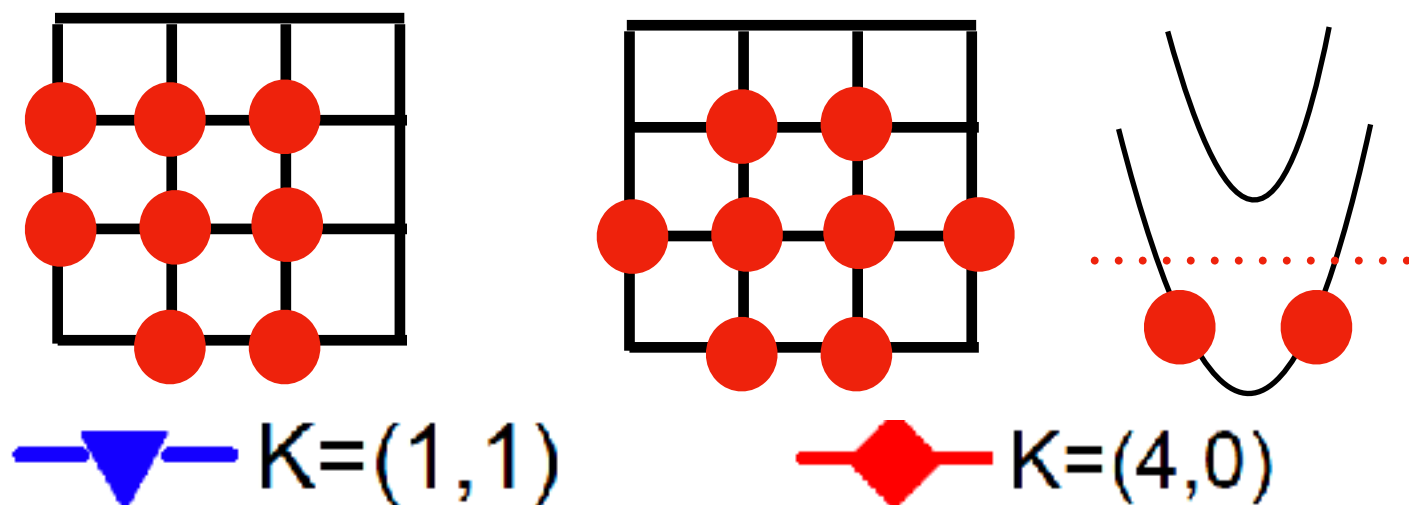
3-fold (x 2) topological degeneracy
of Moore-Read state

	$K=(4,4)$	(π, π)
	$K=(4,0)$	$(\pi, 0)$
	$K=(0,4)$	$(0, \pi)$

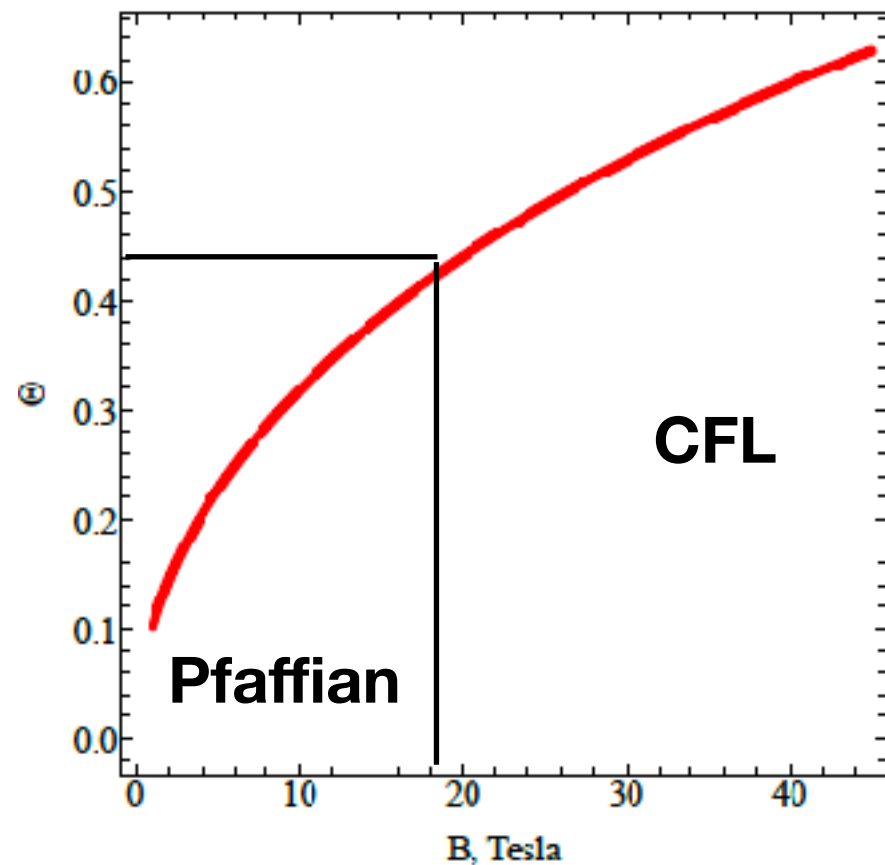


$$S^2 = S_z^{max} (S_z^{max} + 1)$$

$$S^2 = 0$$

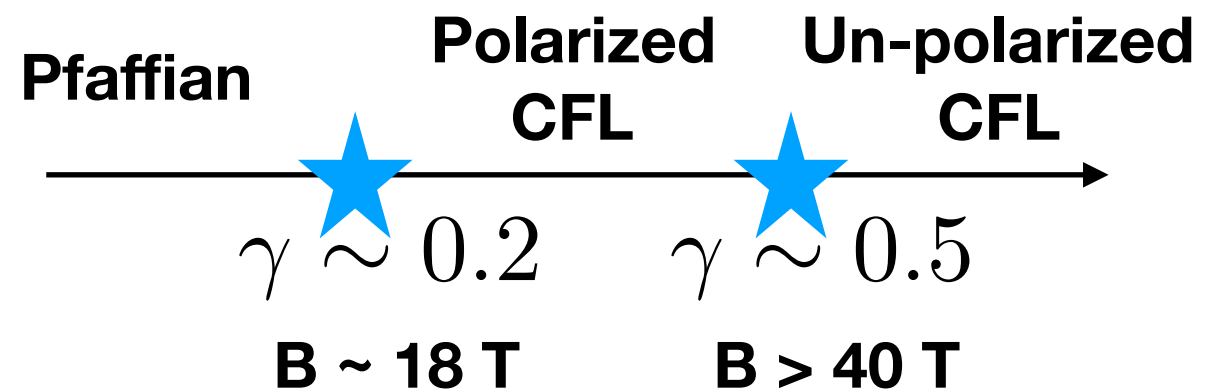


Clean Pfaffian to CFL in bilayer graphene



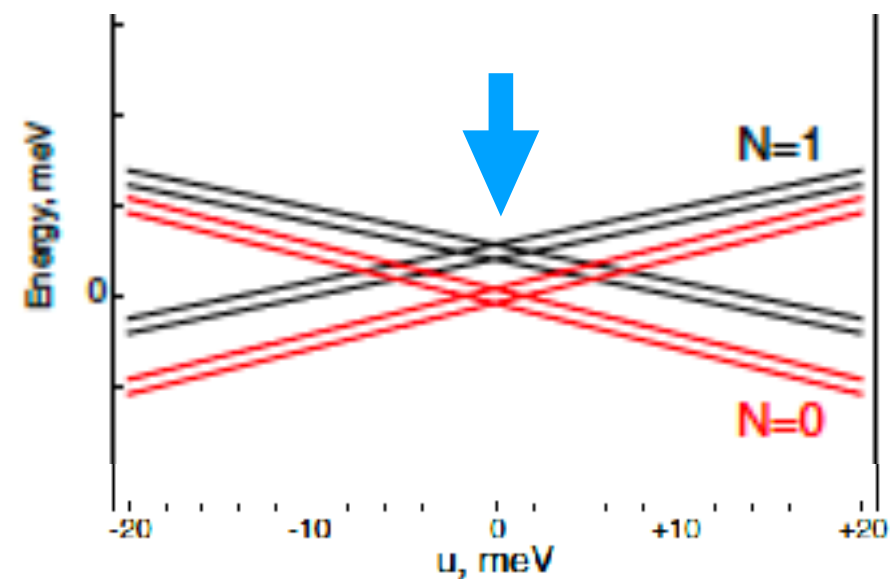
Continuously rotating the $N=0$ LL into a $N=1$ LL with field

$$|\gamma\rangle = \gamma|0, \uparrow\rangle + (1 - \gamma)|1, \downarrow\rangle$$



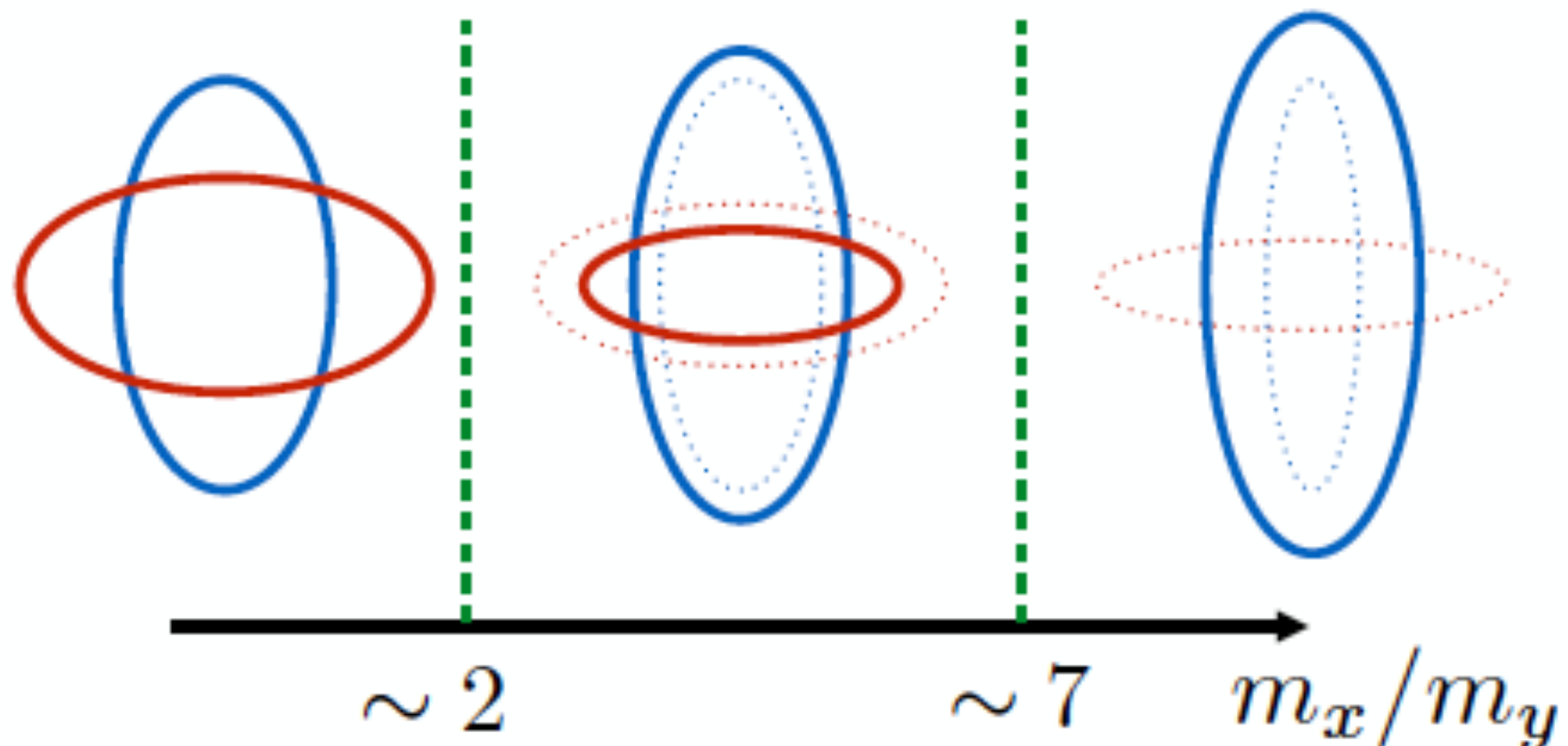
It would be a Pfaffian with a twist

- SU(2) skyrmions and ferromagnet coexisting with Pfaffian physics



Ising Stoner Instability of Composite Fermion Metal

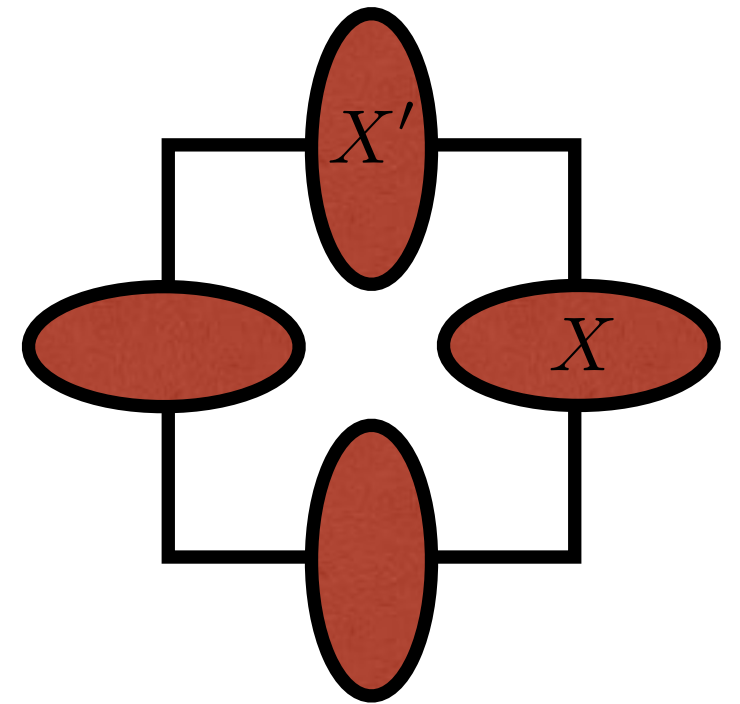
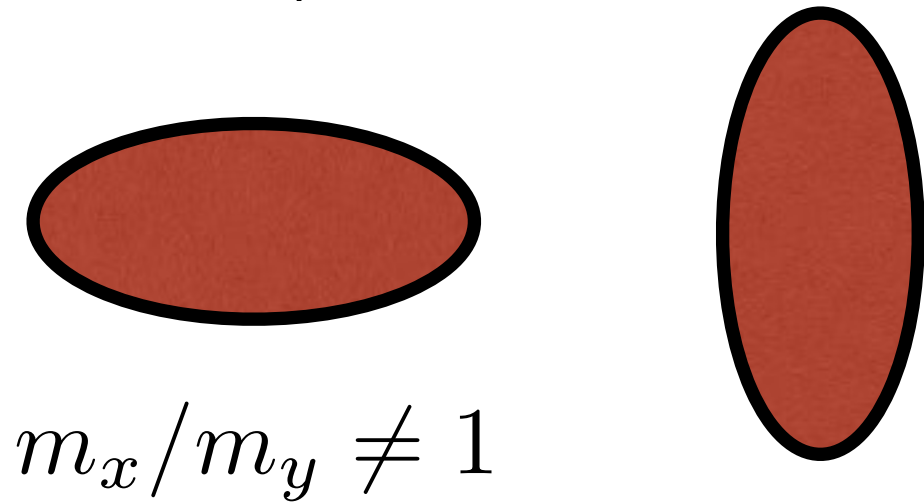
- Two components with rotated mass tensors rotated by $\pi/2$ undergo an analogue of the Stoner transition:



- Aluminum Arsenide $m_x/m_y \approx 5$

The Hamiltonian of our study

Aluminum Arsenide: Two valleys with anisotropic mass



$$\mathbf{p} \rightarrow \mathbf{p} + \mathbf{A}(\mathbf{r}) \quad N = 0LL \quad \begin{array}{c} \text{---} \\ \text{---} \\ X' \\ \text{---} \\ \text{---} \\ X \end{array} \quad \begin{array}{c} \uparrow \\ \downarrow \end{array} \omega_c = \frac{B}{\sqrt{m_x m_y}}$$

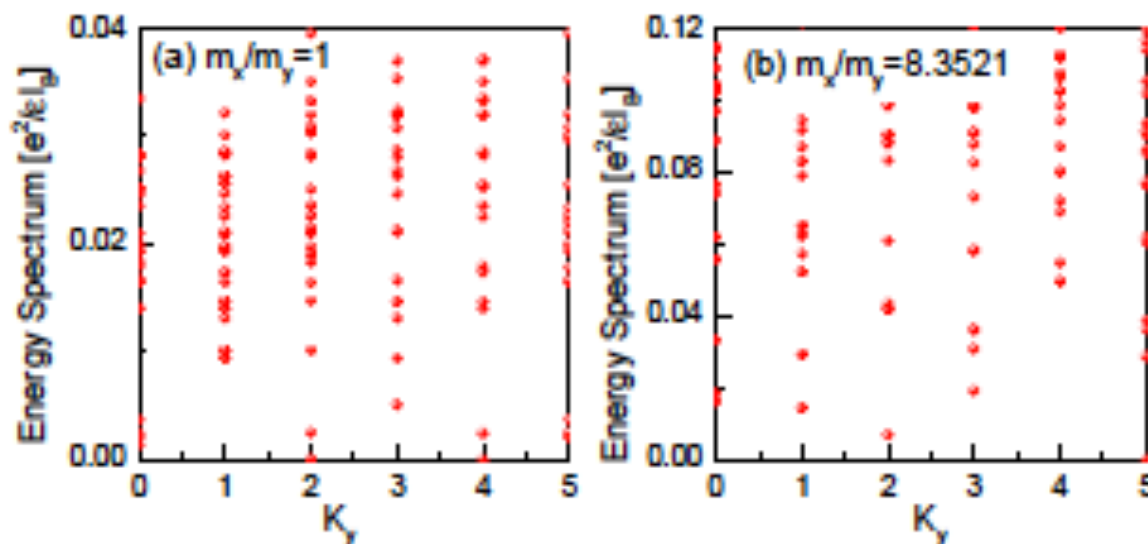
Landau level projection endows electrons with a “shape”

$$v_0(q) = \frac{2\pi e^2}{\epsilon q} \xrightarrow{\text{LL projection}} v_{eff}(q) = \frac{2\pi e^2}{\epsilon q} F_i(q) F_j^*(q)$$

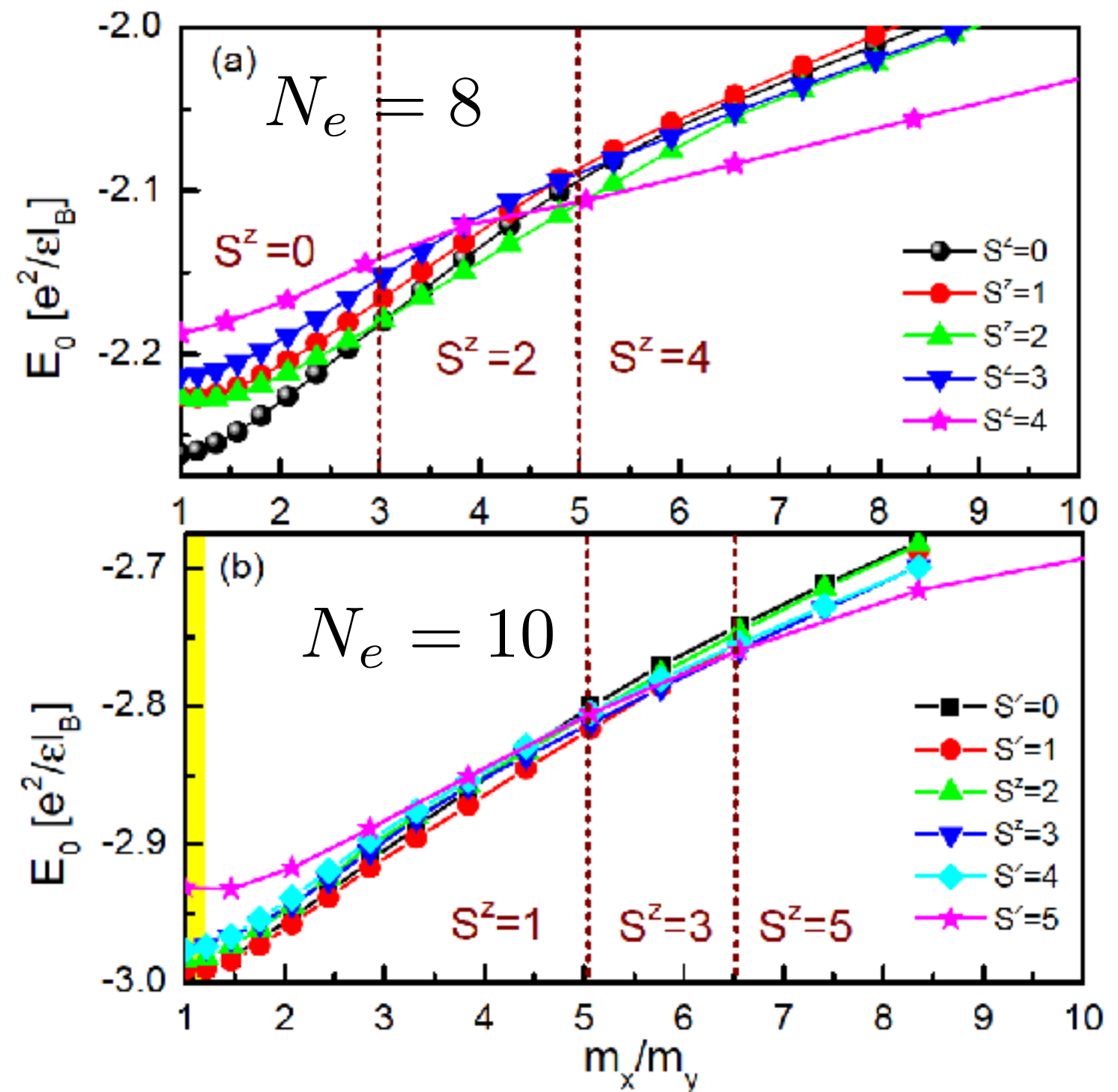
Ground state polarization

$$S^z = \frac{N_1 - N_2}{2}$$

No clear gap in spectrum



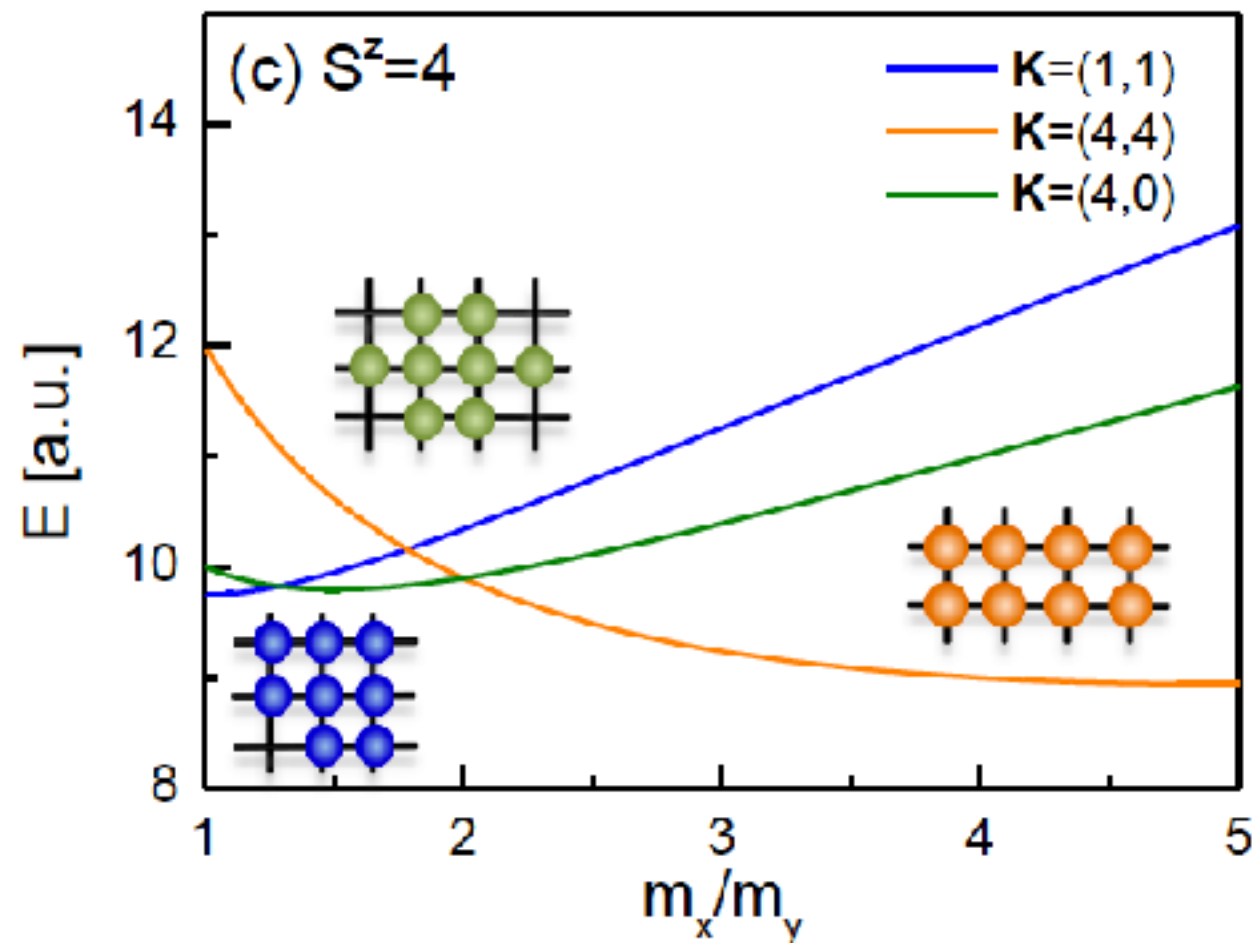
Exact diagonalization on torus



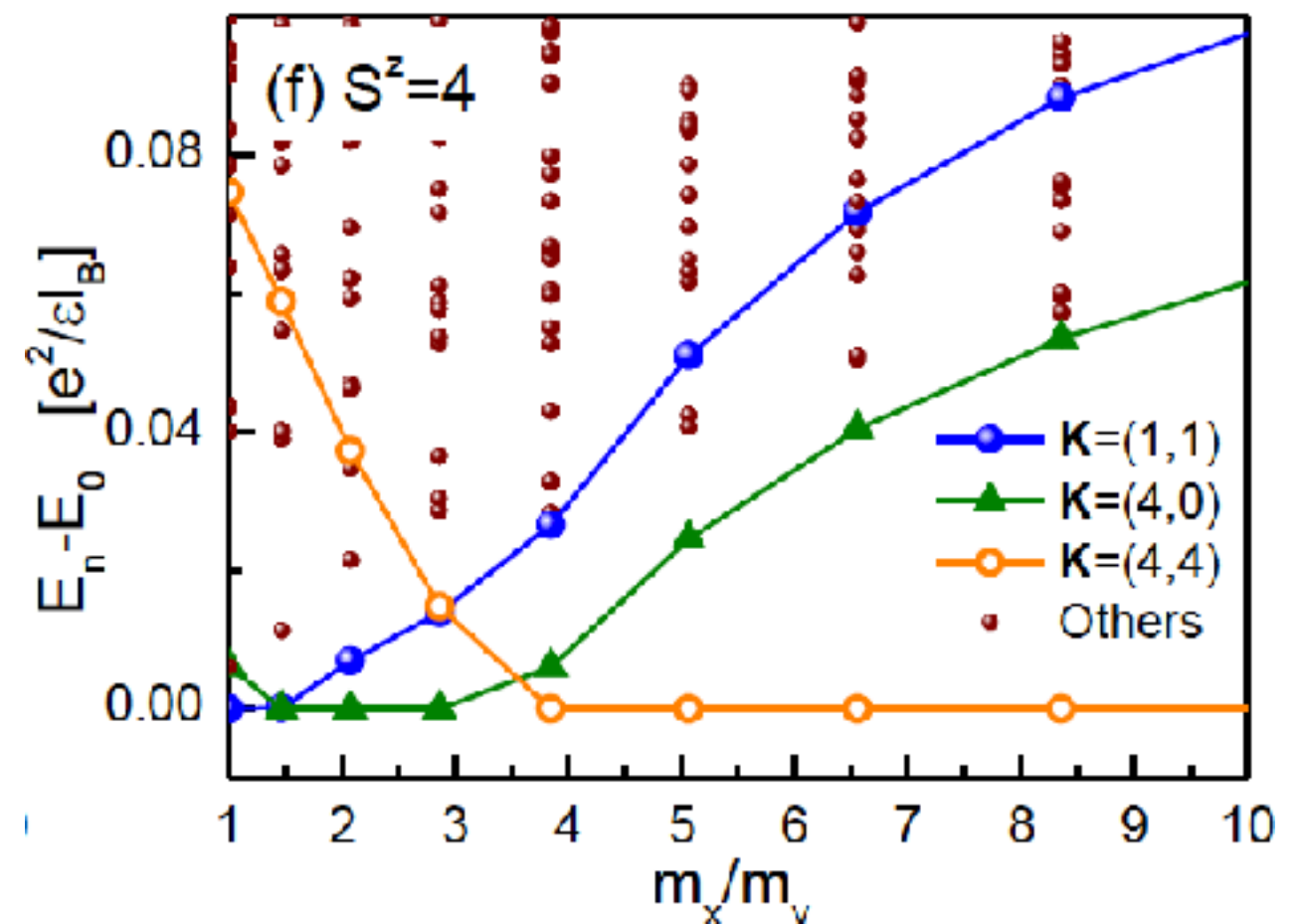
Numerics vs simple model

$$|\Psi_{\text{CFL}}(\{\mathbf{d}_i^\uparrow, \mathbf{d}_i^\downarrow\})\rangle = \det(\hat{t}_j(\mathbf{d}_i^\uparrow)) \det(\hat{t}_j(\mathbf{d}_i^\downarrow)) |\Phi_{1/2}^{\text{Bose}}\rangle$$

Trial wave function

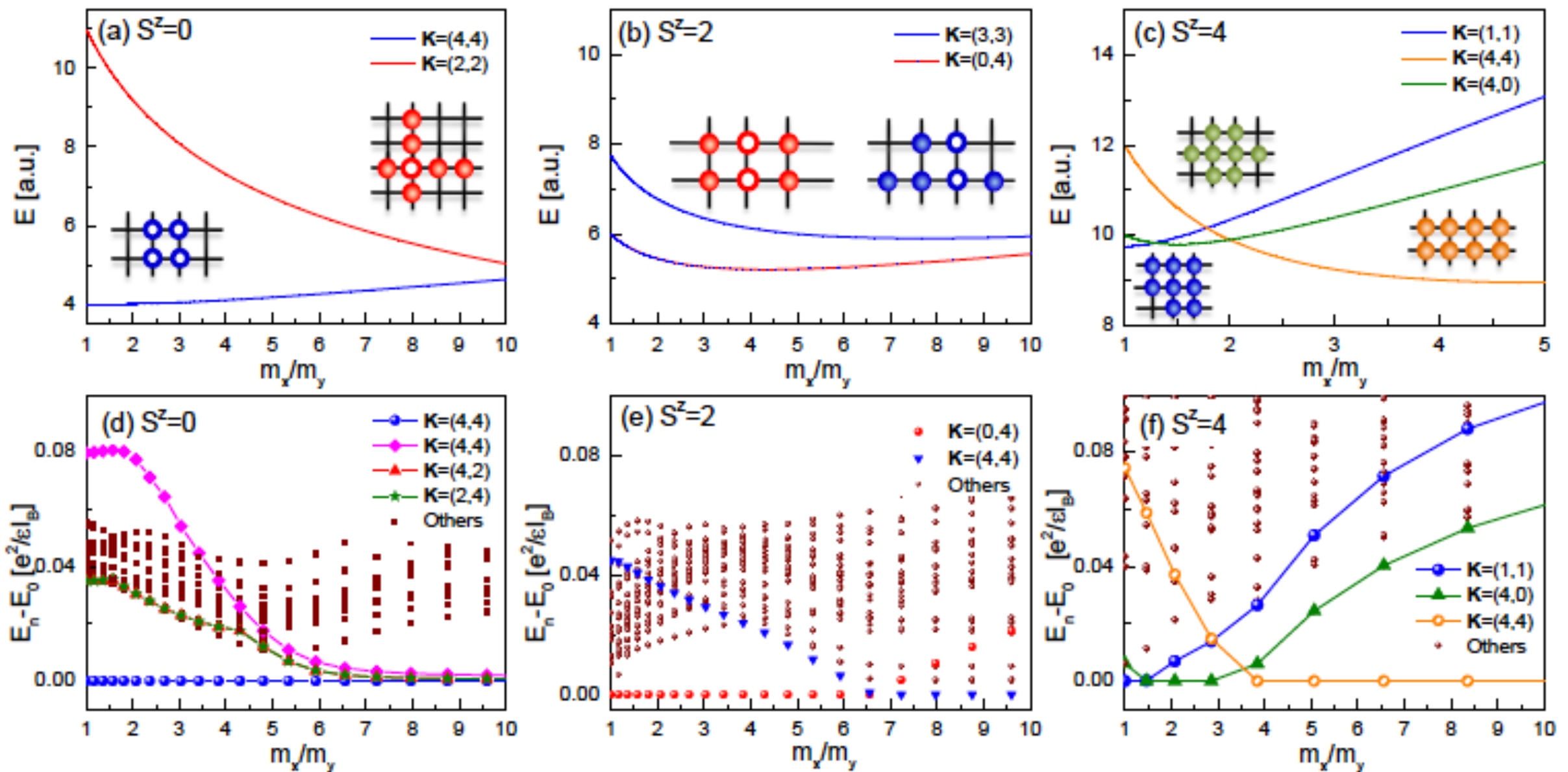


Exact diagonalization



Numerics vs simple model

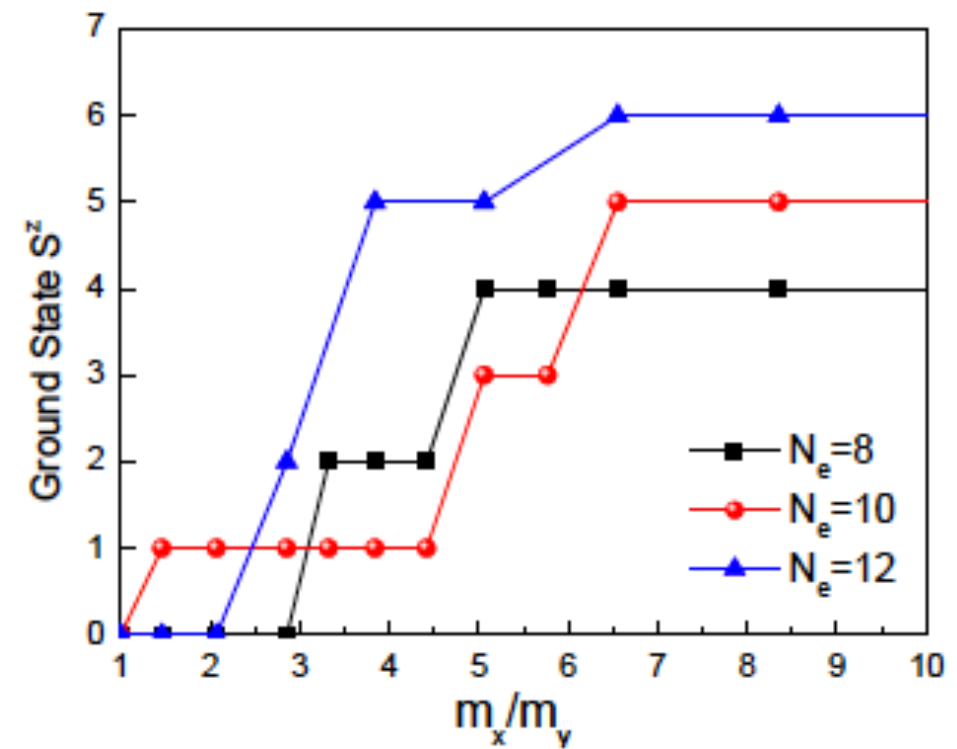
Trial wave function



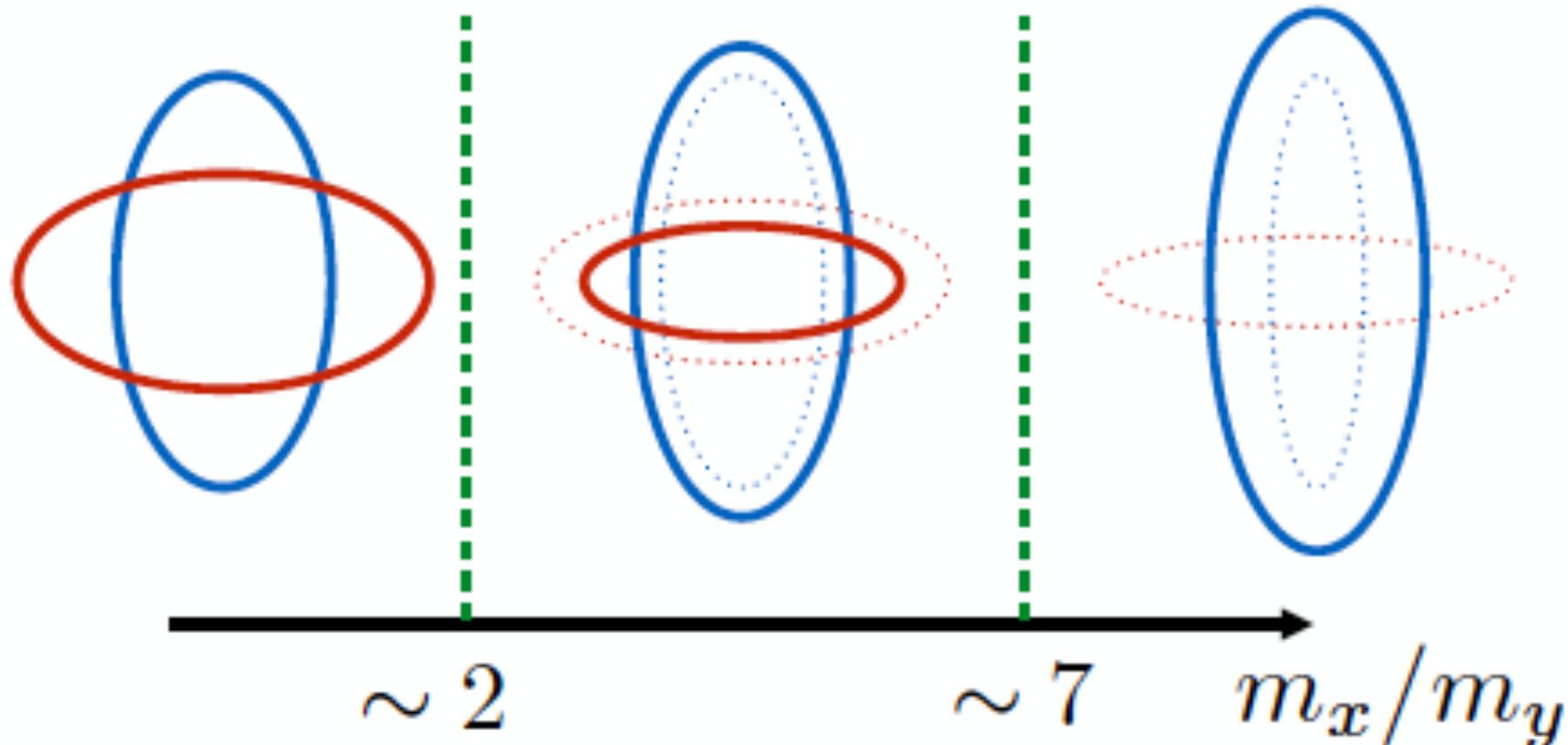
Exact diagonalization

DMRG phase diagram

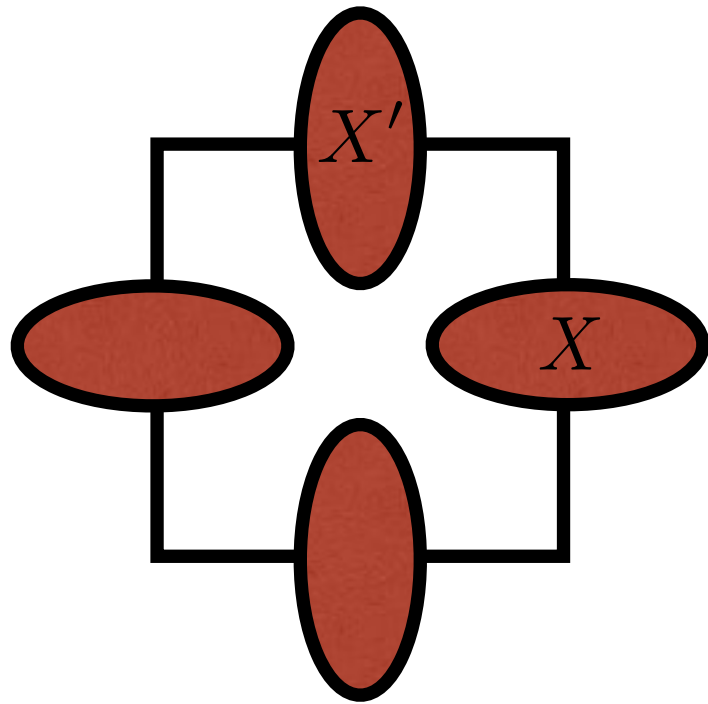
DMRG results:



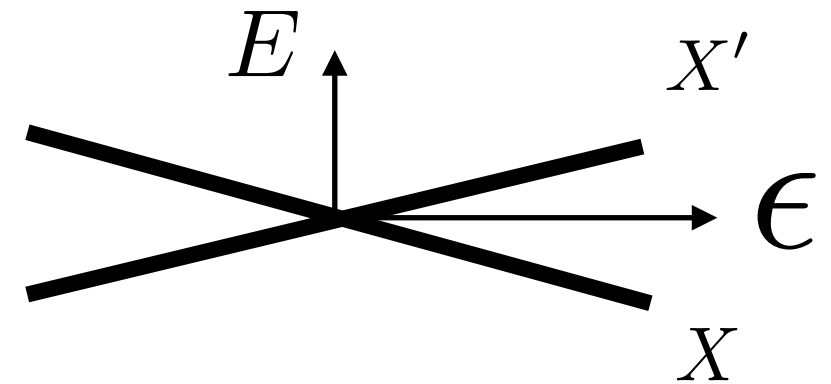
Valley Stoner magnet:



Connections with Aluminum Arsenide

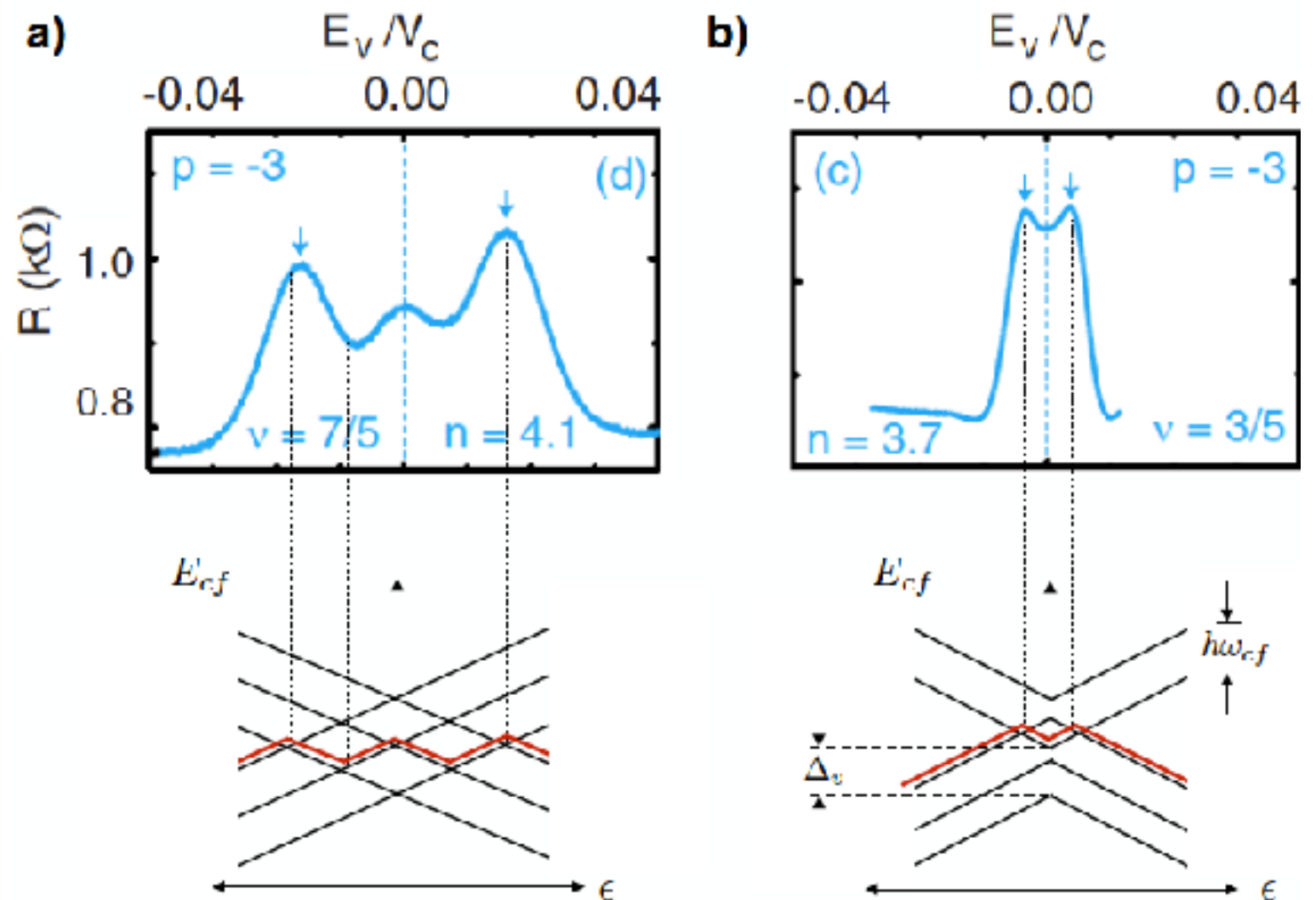


$$N = 0LL$$



$$\nu = 1/2$$

Nearby FQH states appear to have spontaneous valley polarization:



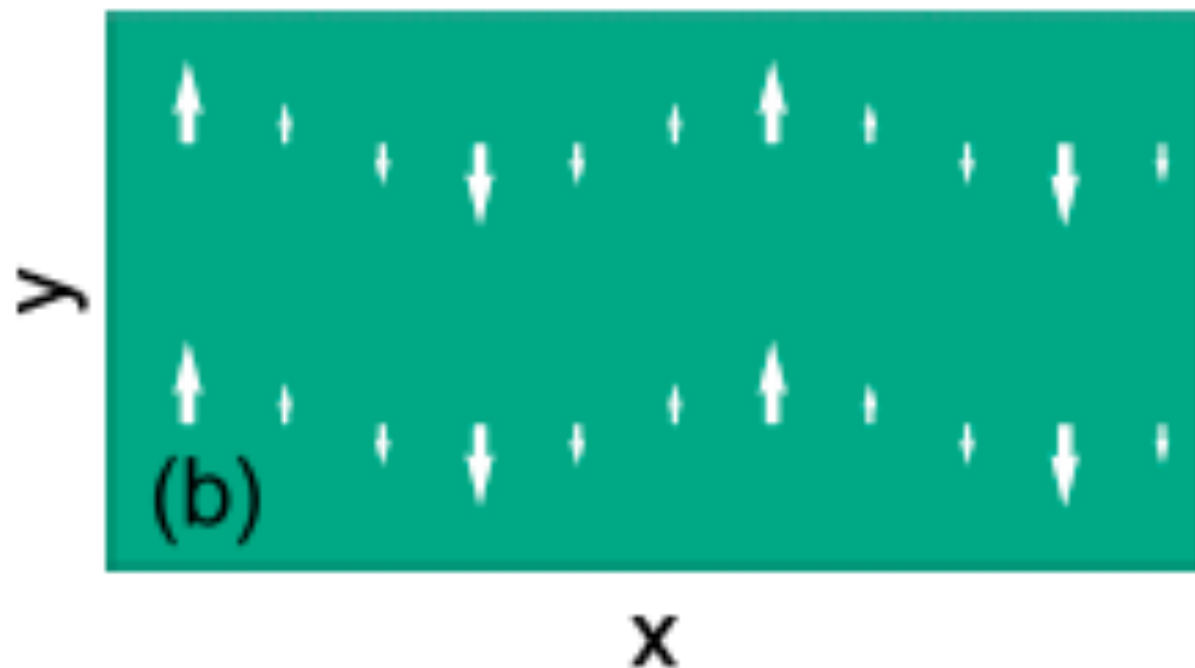
Summary

Alternative platforms for fractional quantum Hall physics like Graphene, ZnO, AIAs, allow to realise a wealth of phase transitions between fractionalized phases of matter.

- Stoner transition of composite fermi liquids in AIAs (arXiv:1802.02167).
- Quantum phase transition between Pfaffian and composite fermi liquid might be realisable in bilayer graphene with perpendicular field.
- Sequence of phase transitions in ZnO (arXiv:1804.04565) and bilayer graphene (Nature 549, 360 (2017)) for a level crossing between $N=0$ and $N=1$ Landau levels.

Bosonization and shear sound in 2D Fermi liquids

Shear sound of interacting Fermi liquids



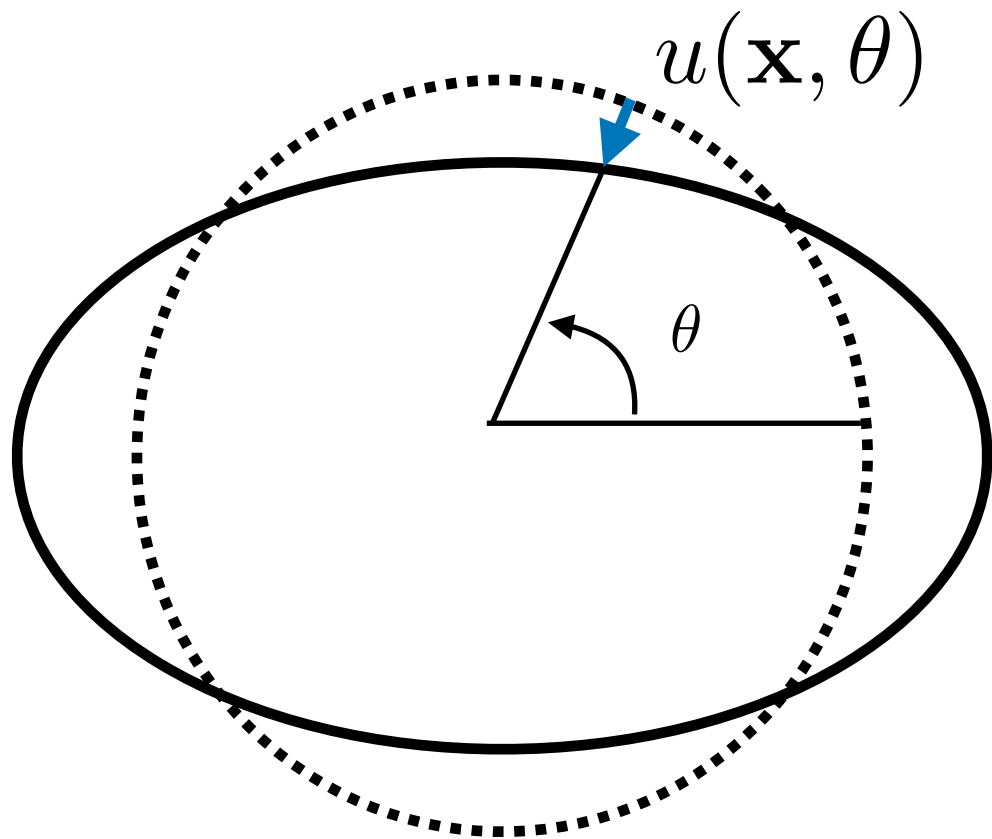
Jun Yong Khoo
MIT

**Should appear rather generically in 2D Fermi liquids
when quasiparticles become twice as heavy as bare**

$$m^* > 2m_0$$

Bosonization of 2D Fermi liquids

2D Fermi liquids have an infinite number of slow variables



State is parametrized by
space-time dependent
Fermi radius

$$p_F(\mathbf{x}, \theta) = p_{F0} + u(\mathbf{x}, \theta)$$

Radius has commutation relations analogous to 1D

$$\partial_n = \hat{\mathbf{p}}_\theta \cdot \partial_{\mathbf{x}}$$

$$[\hat{u}_{\mathbf{x}, \theta}, \hat{u}_{\mathbf{x}', \theta'}] = \frac{(2\pi)^2}{ip_F} \delta(\theta - \theta') \partial_n \delta(\mathbf{x} - \mathbf{x}') + O(\hat{u})$$

A. Luther, *Phys. Rev. B* **19**, 320 (1979).

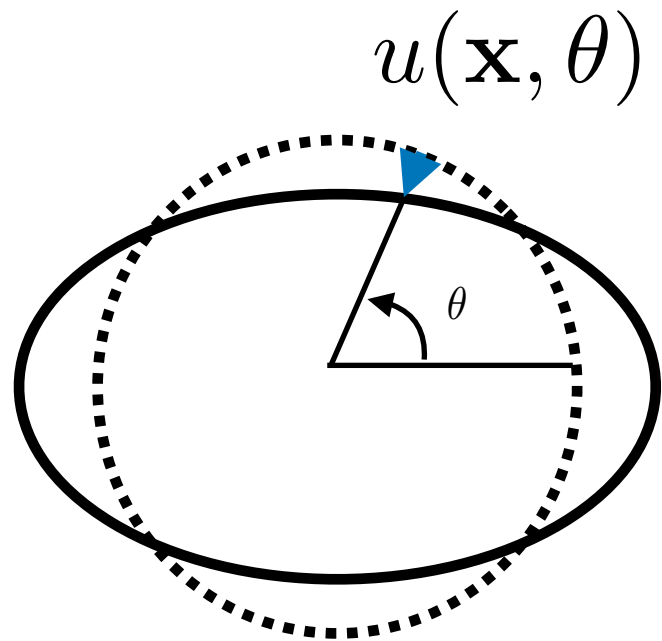
F. D. M. Haldane, *eprint arXiv:cond-mat/0505529*

A. H. Castro Neto and E. Fradkin, *Phys. Rev. Lett.* **72**, 1393 (1994).

D. F. Mross and T. Senthil, *Phys. Rev. B* **84**, 165126 (2011).

S. Golkar, D. X. Nguyen, M. M. Roberts, and D. T. Son, *Phys. Rev. Lett.* **117**, 216403 (2016).

Bosonization of 2D Fermi liquids



$$\partial_n = \hat{\mathbf{p}}_\theta \cdot \partial_{\mathbf{x}}$$

$$[\hat{u}_{\mathbf{x},\theta}, \hat{u}_{\mathbf{x}',\theta'}] = \frac{(2\pi)^2}{ip_F} \delta(\theta - \theta') \partial_n \delta(\mathbf{x} - \mathbf{x}') + O(\hat{u})$$

$$\hat{H} = \int d^2\mathbf{x} \hat{u}_{\mathbf{x},\theta}^\dagger h_{\theta,\theta'} \hat{u}_{\mathbf{x},\theta'}$$

$$h_{\theta,\theta'} = \frac{v_F p_F}{2} \left(\delta(\theta' - \theta) + \frac{F(\theta' - \theta)}{2\pi} \right)$$

Landau function:
 $F(\theta' - \theta)$

Quantum version of kinetic equation:

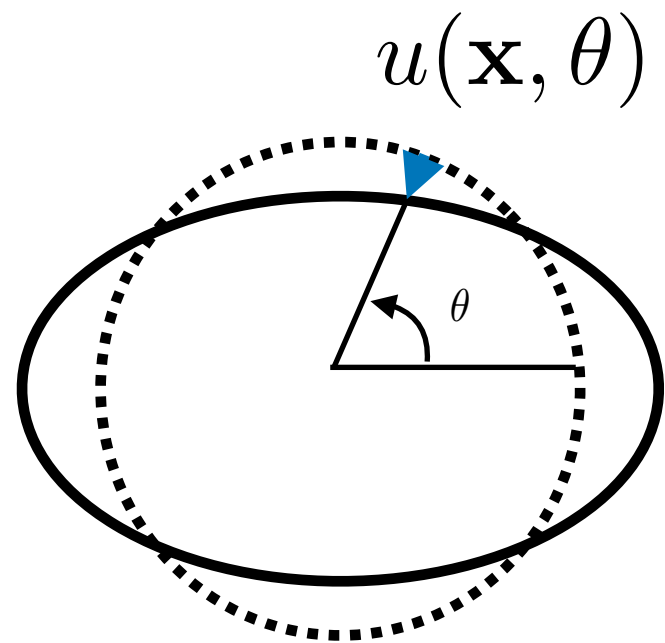
$$i\partial_t \hat{u}_{\mathbf{q},\theta} = [\hat{u}_{\mathbf{q},\theta}, \hat{H}] = K_{\theta,\theta'} \hat{u}_{\mathbf{q},\theta'},$$

$$\hat{u}_{\mathbf{q},\theta} \equiv \int d^2\mathbf{x} \hat{u}_{\mathbf{x},\theta} e^{-i\mathbf{q}\cdot\mathbf{x}}$$

$$K(\theta, \theta') = v_F \mathbf{q} \cdot \hat{\mathbf{p}}_\theta \left(\delta(\theta - \theta') + \frac{1}{2\pi} F(\theta - \theta') \right)$$

$$v_\theta^\dagger G_{\theta,\theta'} w_{\theta'} \equiv \int \frac{d\theta d\theta'}{(2\pi)^2} v(\theta) G(\theta, \theta') w(\theta')$$

Mapping classical to quantum



Quantum version of kinetic equation:

$$i\partial_t \hat{u}_{\mathbf{q},\theta} = [\hat{u}_{\mathbf{q},\theta}, \hat{H}] = K_{\theta,\theta'} \hat{u}_{\mathbf{q},\theta'},$$

$$K(\theta, \theta') = v_F \mathbf{q} \cdot \hat{\mathbf{p}}_\theta \left(\delta(\theta - \theta') + \frac{1}{2\pi} F(\theta - \theta') \right)$$

$$K = TK^\dagger T^{-1}, \quad T_{\theta,\theta'} = \frac{(2\pi)^2 \mathbf{q} \cdot \hat{\mathbf{p}}_\theta}{p_F} \delta(\theta - \theta')$$

For every classical eigen-function of the kinetic Equation:

$$K_{\theta,\theta'} \psi_{\lambda,\mathbf{q},\theta'} = E_\lambda \psi_{\lambda,\mathbf{q},\theta}$$

We can construct a quantum bosonic eigen-mode:

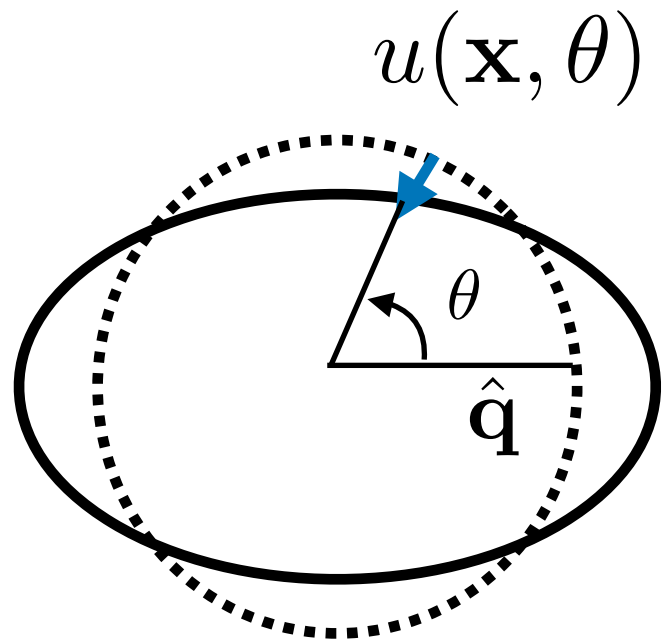
$$\hat{\psi}_{\lambda,\mathbf{q}} = \psi_{\lambda,\mathbf{q},\theta}^\dagger T_{\theta,\theta'}^{-1} \hat{u}_{\mathbf{q},\theta'}$$

Canonical normalisation:

$$\psi_{\lambda,\mathbf{q},\theta}^\dagger T_{\theta,\theta'}^{-1} \psi_{\lambda',\mathbf{q},\theta'} = \text{sgn}(E_\lambda) \delta_{\lambda,\lambda'}$$

$$v_\theta^\dagger G_{\theta,\theta'} w_{\theta'} \equiv \int \frac{d\theta d\theta'}{(2\pi)^2} v(\theta) G(\theta, \theta') w(\theta')$$

Mapping classical to 1D tight binding



Mirror symmetry:

$$F(\theta) = F(-\theta), \quad K_{\theta, \theta'} = K_{-\theta, -\theta'}$$

Even and Odd modes: $\sigma = \pm$

$$\psi_{\lambda, \mathbf{q}, \theta}^{\sigma} = \sigma \psi_{\lambda, \mathbf{q}, -\theta}^{\sigma}$$

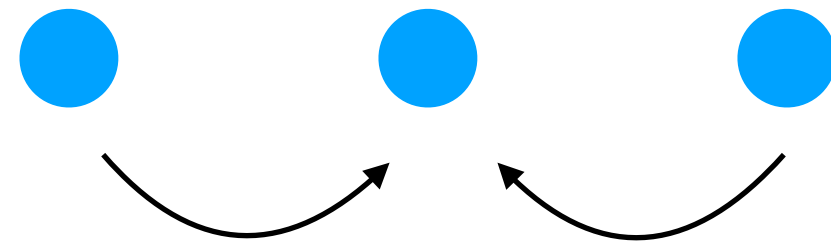
Angular momentum:

$$F(\theta) = F_0 + \sum_{l=1}^{\infty} 2F_l \cos(l\theta)$$

$$\psi_{\lambda, \theta}^{+} = \psi_{\lambda, 0}^{+} + \sum_{l=1}^{\infty} 2\psi_{\lambda, l}^{+} \cos(l\theta),$$

$$\psi_{\lambda, \theta}^{-} = \sum_{l=1}^{\infty} 2\psi_{\lambda, l}^{-} \sin(l\theta).$$

$$E_{\lambda}^{\sigma} \psi_{\lambda, l+1}^{\sigma} = t_l \psi_{\lambda, l}^{\sigma} + t_{l+2} \psi_{\lambda, l+2}^{\sigma}$$



$$t_l = v_F q (1 + F_l) / 2$$

Mapping classical to 1D tight binding

Angular momentum:

$$F(\theta) = F_0 + \sum_{l=1}^{\infty} 2F_l \cos(l\theta)$$

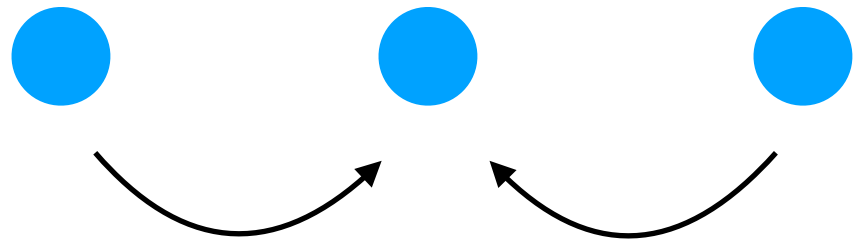
$$\psi_{\lambda,\theta}^+ = \psi_{\lambda,0}^+ + \sum_{l=1}^{\infty} 2\psi_{\lambda,l}^+ \cos(l\theta),$$

$$\psi_{\lambda,\theta}^- = \sum_{l=1}^{\infty} 2\psi_{\lambda,l}^- \sin(l\theta).$$

Even and Odd modes: $\sigma = \pm$

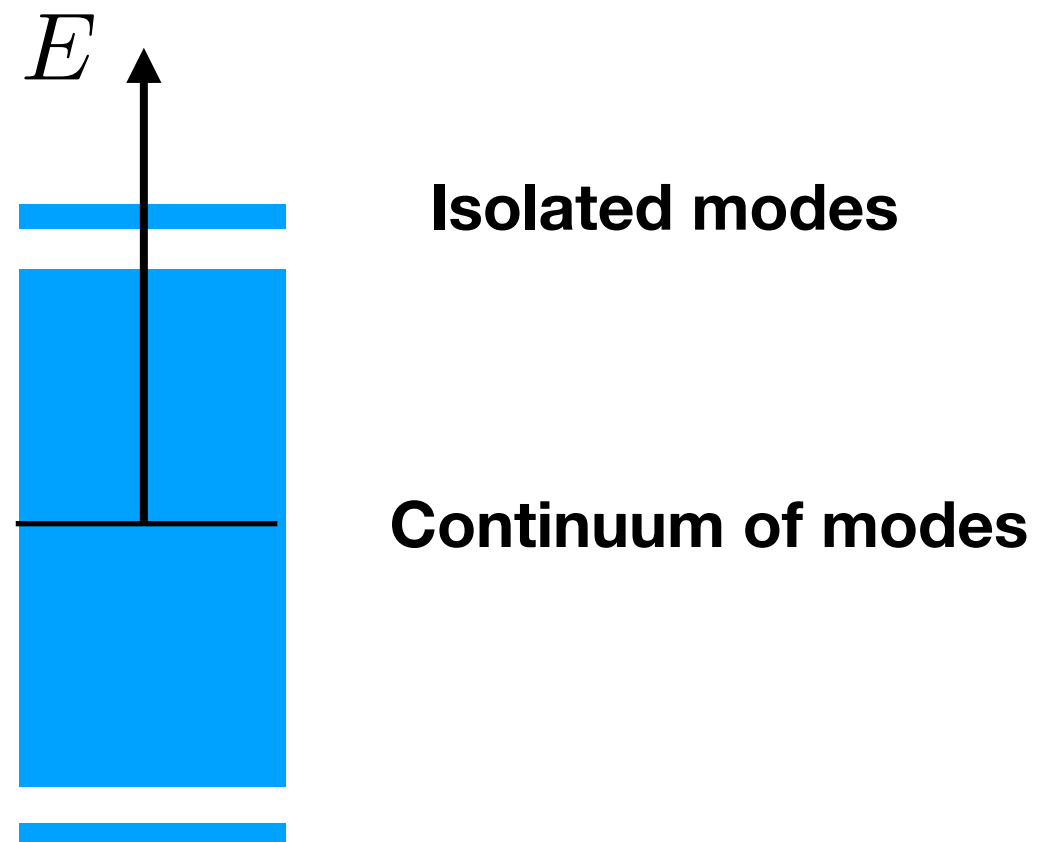
$$\psi_{\lambda,\mathbf{q},\theta}^{\sigma} = \sigma \psi_{\lambda,\mathbf{q},-\theta}^{\sigma}$$

$$E_{\lambda}^{\sigma} \psi_{\lambda,l+1}^{\sigma} = t_l \psi_{\lambda,l}^{\sigma} + t_{l+2} \psi_{\lambda,l+2}^{\sigma}$$



$$t_l = v_F q (1 + F_l) / 2$$

Landau parameters
play role of bond disorder

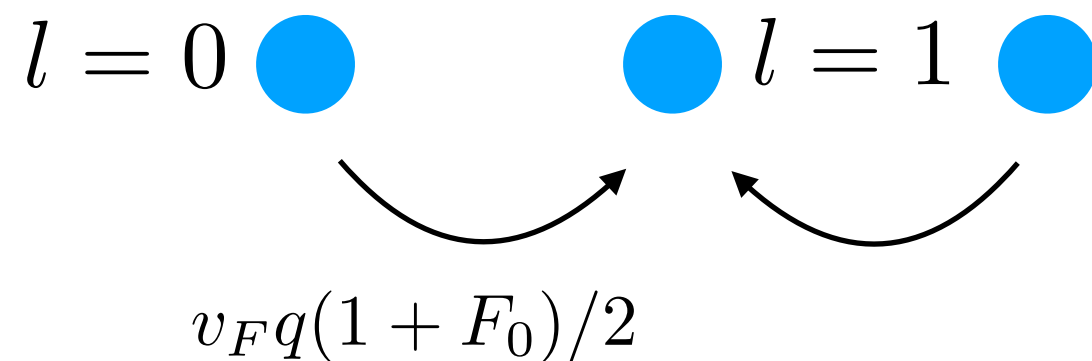


Another sound

Only one bond is defective:

$$F_0 > 0 \quad F_{l>0} = 0$$

$$E_\lambda^\sigma \psi_{\lambda,l+1}^\sigma = t_l \psi_{\lambda,l}^\sigma + t_{l+2} \psi_{\lambda,l+2}^\sigma$$



$$t_l = v_F q(1 + F_l)/2$$

Mapping between even and odd sector problems

$$l \rightarrow l + 1$$

$$F'_{l+1} = F_l \quad \text{for } l \geq 1$$

$$F'_1 = 1 + 2F_0$$

Even parity modes

$$l = 0 \quad l = 1 \quad \dots$$



Odd parity modes



$$l = 1$$

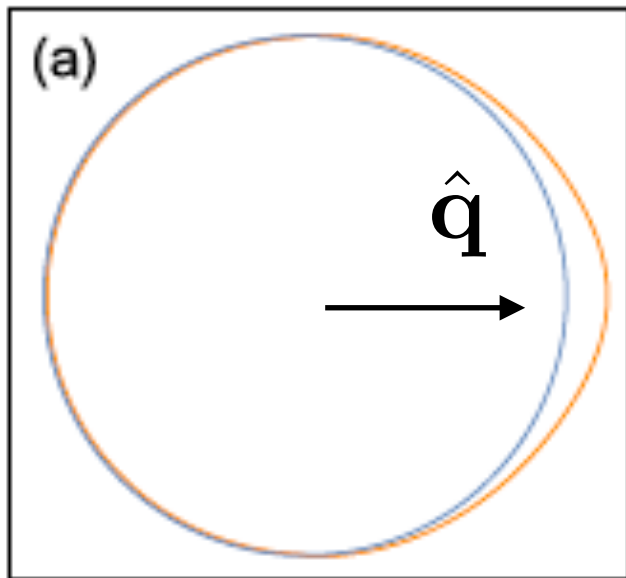
$$l = 2$$

$\dots$

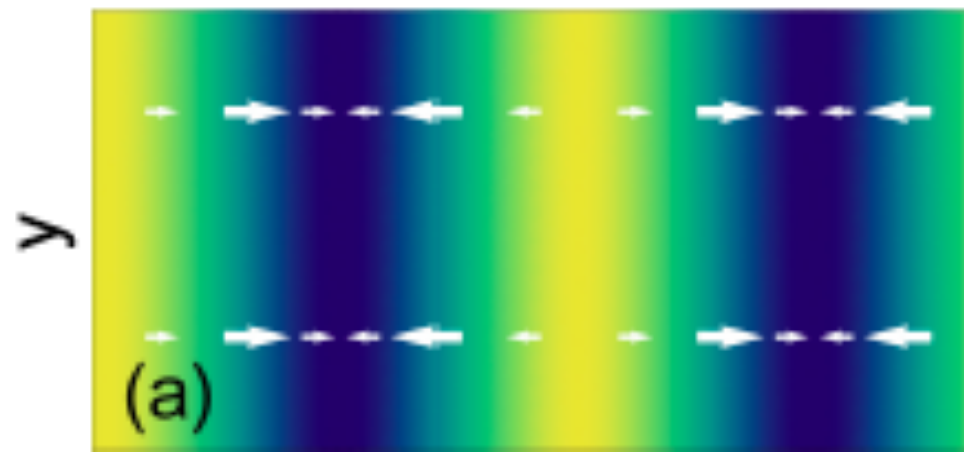
Implies existence of collective mode other than zero sound in model with non-zero:

$$\{F_0, F_1\}$$

Shear vs zero sound



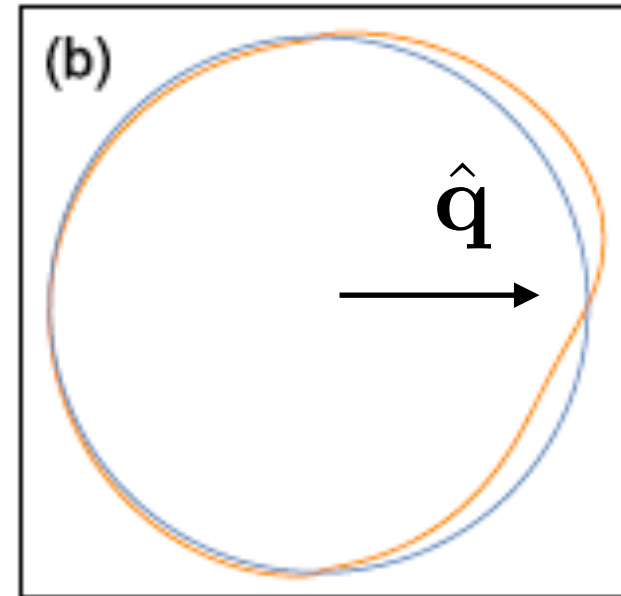
Zero sound is longitudinal



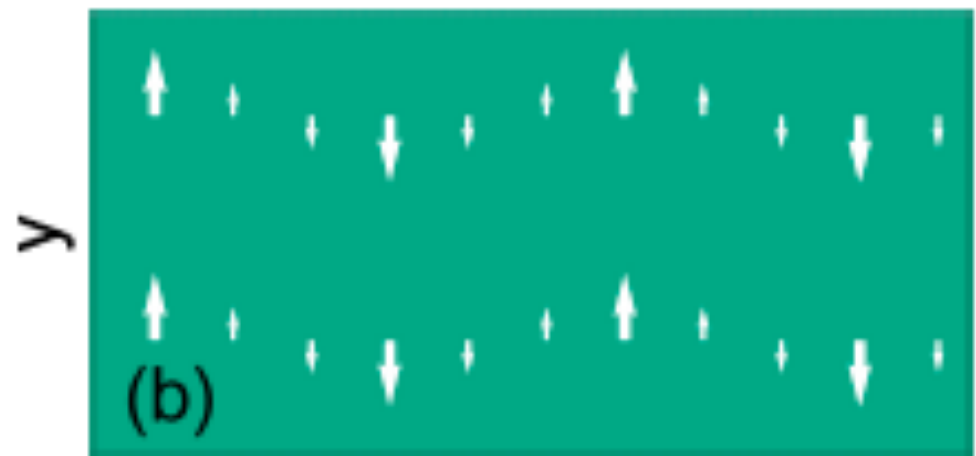
$$\mathbf{j} \parallel \mathbf{q}$$

In metals zero sound is transformed into plasma mode

$$E \propto \sqrt{|\mathbf{q}|}$$



Shear sound is purely transverse



$$\mathbf{j} \perp \mathbf{q}$$

Shear sound remains linearly dispersing in metals

$$E = v_{\perp} |\mathbf{q}|$$

Shear sound and mass renormalization

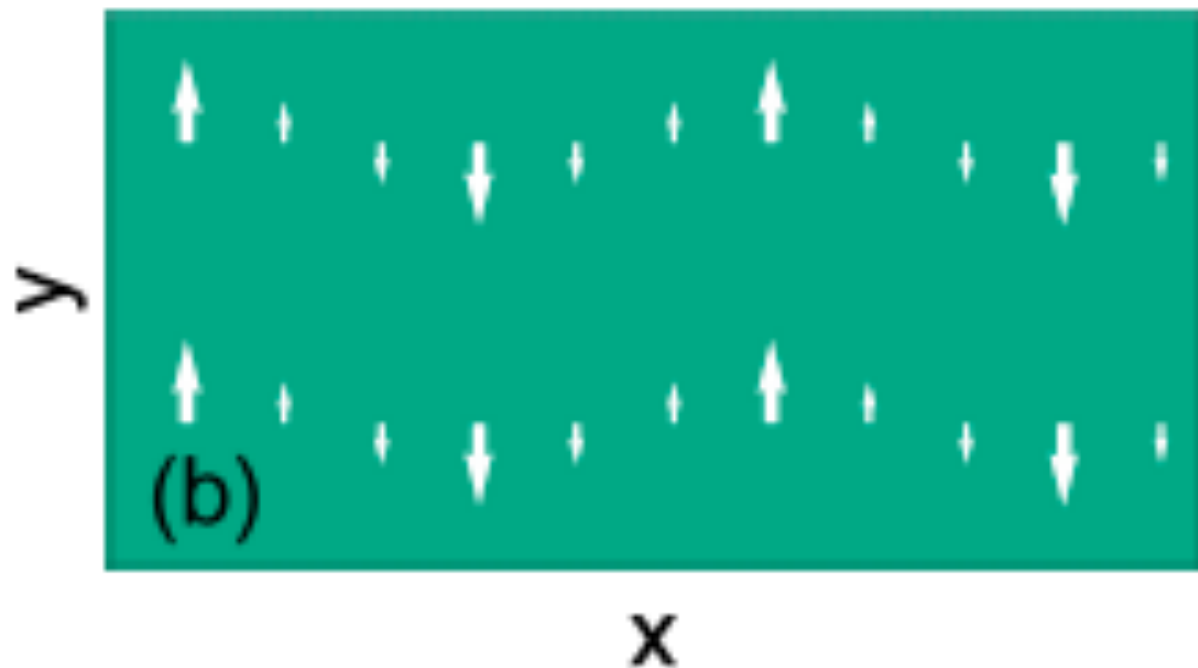
Mode is Landau damped in weakly interacting Fermi liquids

Mode is expected to appear out of continuum when:

$$F_1 > 1$$

Mode is expected to appear when quasiparticles become twice as heavy:

$$\frac{v_{F0}}{v_F} = \frac{m^*}{m_0} = 1 + F_1$$



Large variety of systems with mass enhancement near critical points could host undamped shear sound:

- He3 adsorbed on graphite
- kappa-ET and dmit on the pressure induced metal side
- Quasi-2D heavy fermion materials (e.g. CeCoIn5)
- Overdoped metal in iron based superconductors (cuprates?)

M. Neumann, J. Nyéki, B. Cowan, and J. Saunders, *Science* **317**, 1356 (2007).

Y. Zhou, K. Kanoda, and T.-K. Ng, *Rev. Mod. Phys.* **89**, 025003 (2017).

Settai et al. *Journal of Physics: Condensed Matter* **13**, L627 (2001)

Hashimoto et al. *Science* **336**, 1554 (2012).

Response functions

		Density	Longitudinal Current
$\hat{O}_{\mathbf{q}} = \int d\theta O(\mathbf{q}, \theta) \hat{u}_{\mathbf{q}, \theta} = \sum_{\lambda} O_{\lambda, \mathbf{q}} \hat{\psi}_{\lambda, \mathbf{q}}$			
$\rho_{\lambda, \mathbf{q}} = \text{sgn}(E_{\lambda}) \frac{p_F}{2\pi} \psi_{\lambda, 0}^+$	Even Modes	$l = 0$ 	$l = 1$ 
$\mathbf{j}_{\lambda, \mathbf{q}} = \text{sgn}(E_{\lambda}) \frac{v_{0F} p_F}{2\pi} (\psi_{\lambda, 1}^+ \hat{\mathbf{q}} + \psi_{\lambda, 1}^- \hat{\mathbf{q}}_{\perp})$			
		Odd modes	Transverse Current  $l = 1$

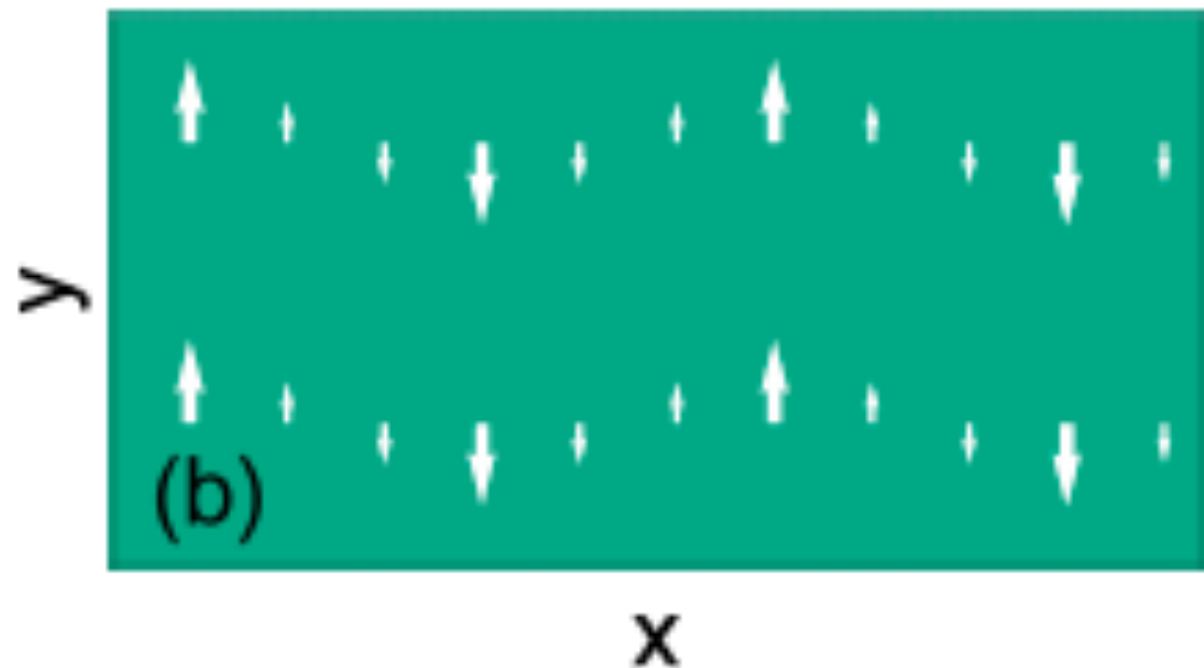
Sharp peak in the transverse current-current correlation function with spectral weight:

$$w_{j_{\perp} j_{\perp}} = \frac{p_F q v_{0F}^2}{16} \frac{F_1 - 1}{F_1^{3/2}} \quad \text{vanishes as } F_1 \rightarrow 1$$

$$\text{Im} \chi_{j_{\perp} j_{\perp}}(\mathbf{q}, \omega) = -\mathcal{A} \frac{\pi v_{0F}^2 p_F^2}{(2\pi)^2} \sum_i |\psi_{i, 1}^-|^2 \text{sgn}(E_i) \delta(\omega - E_i^-)$$

Summary

Mode is Landau damped in weakly interacting Fermi liquids



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Stoner

