

Spinon magnetic resonance

Oleg Starykh, University of Utah



Topological phases of matter [TOPMAT]: from the quantum Hall effect to spin liquids

June 11, 2018

11 Jun-6 Jul 2018 Saclay (France)

My other projects I'd be glad to discuss at the workshop

- Topological phase of repulsively interacting quantum wire with SO coupling
- Weakly coupled critical SU(3) chains [bilinear-biquadratic spin $S=1$ chains]
- Chiral liquid in spin-1 XXZ model on triangular lattice/ $M=1/3$ magnetization plateau

The big question(s)

What is quantum spin liquid?

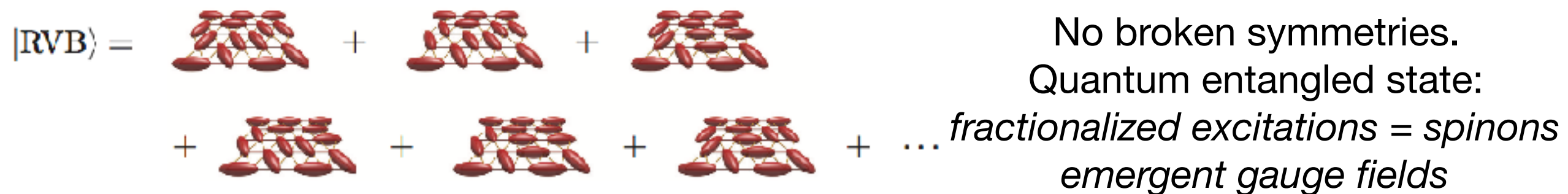


Figure 1. A 'resonating valence bond' (RVB) state. Ellipsoids indicate spin-zero singlet states of two $S = 1/2$ spins.

Savary, Balents 2017

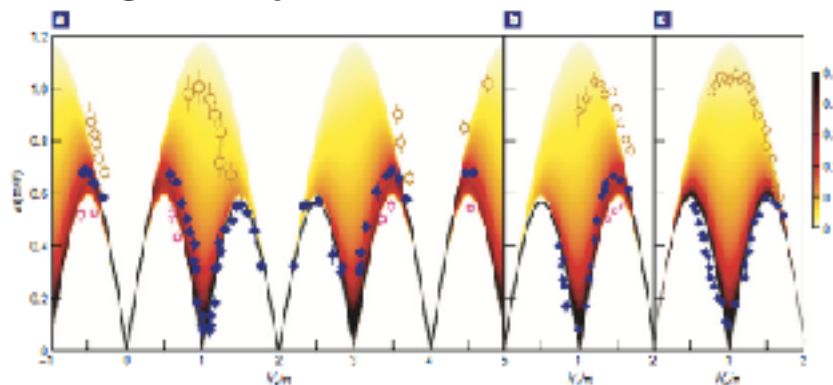
Which materials realize it?

Past candidates: Cs_2CuCl_4 , kagome volborthite...

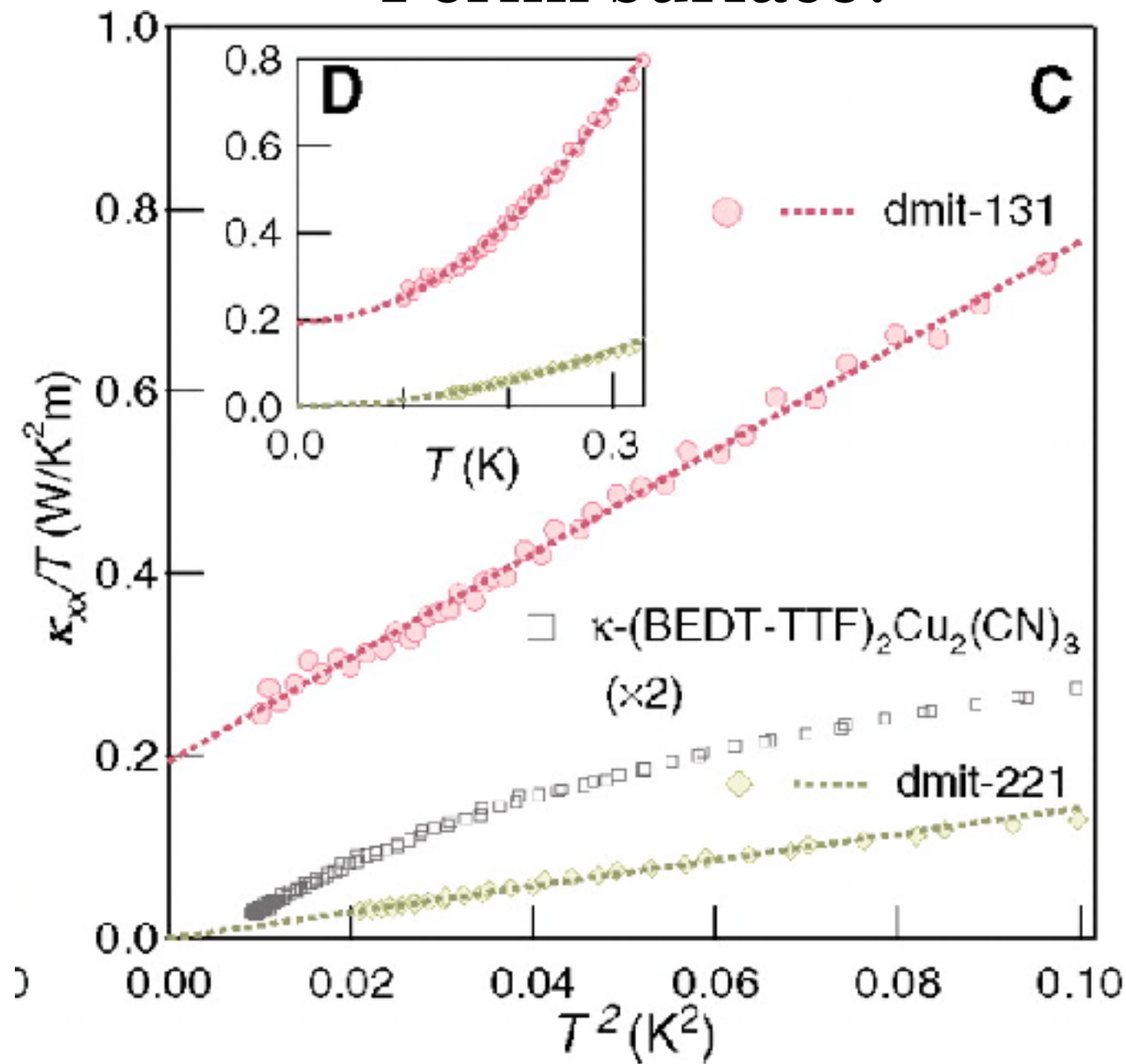
Current candidates: kagome herbertsmithite, $\alpha\text{-RuCl}_3$, organic Mott insulators

How to detect/observe it?

Neutrons (if good single crystals are available), NMR, ESR



Organic Mott insulators: Spin liquid with spinon Fermi surface?



EtMe₃Sb[Pd(dmit)₂]₂ (dmit-131)

Spin liquid?

M. Yamashita et al, Science 2010

Et₂Me₂Sb[Pd(dmit)₂]₂ (dmit-221)

Non-magnetic
charge-ordered

theory: O. Motrunich 2005,
S.-S. Lee and P. A. Lee 2005

electrical insulator,
but metal-like thermal conductor

α -RuCl₃: quantized thermal Hall

Majorana quantization and half-integer thermal quantum Hall effect in a Kitaev spin liquid

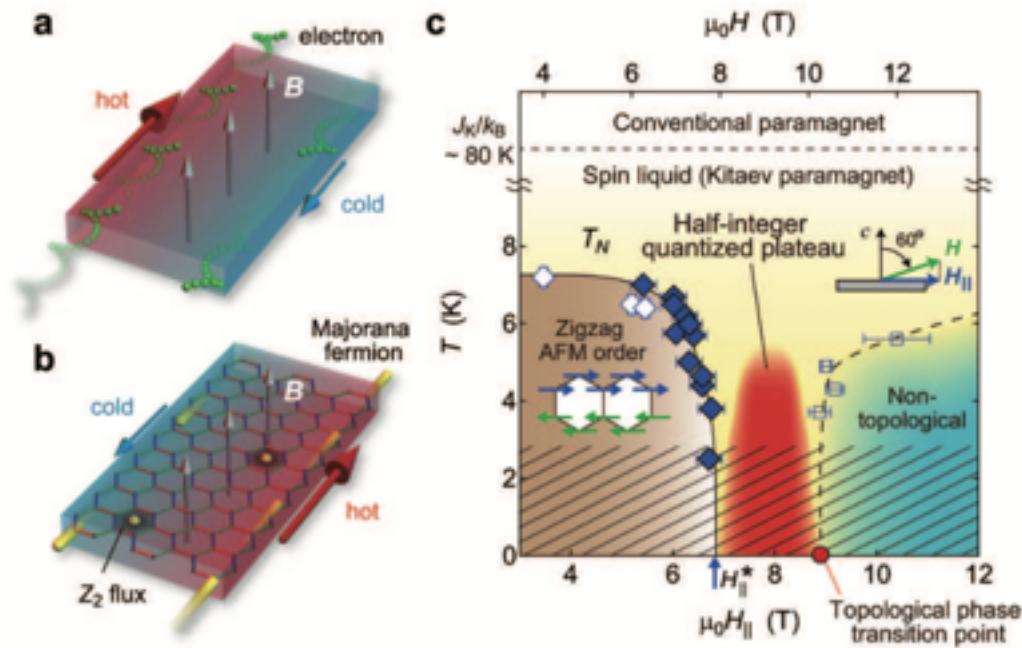
Y. Kasahara¹, T. Ohnishi¹, N. Kurita², H. Tanaka², J. Nasu², Y. Motome³, T. Shibauchi⁴, and Y. Matsuda¹

¹Department of Physics, Kyoto University, Kyoto 606-8502, Japan

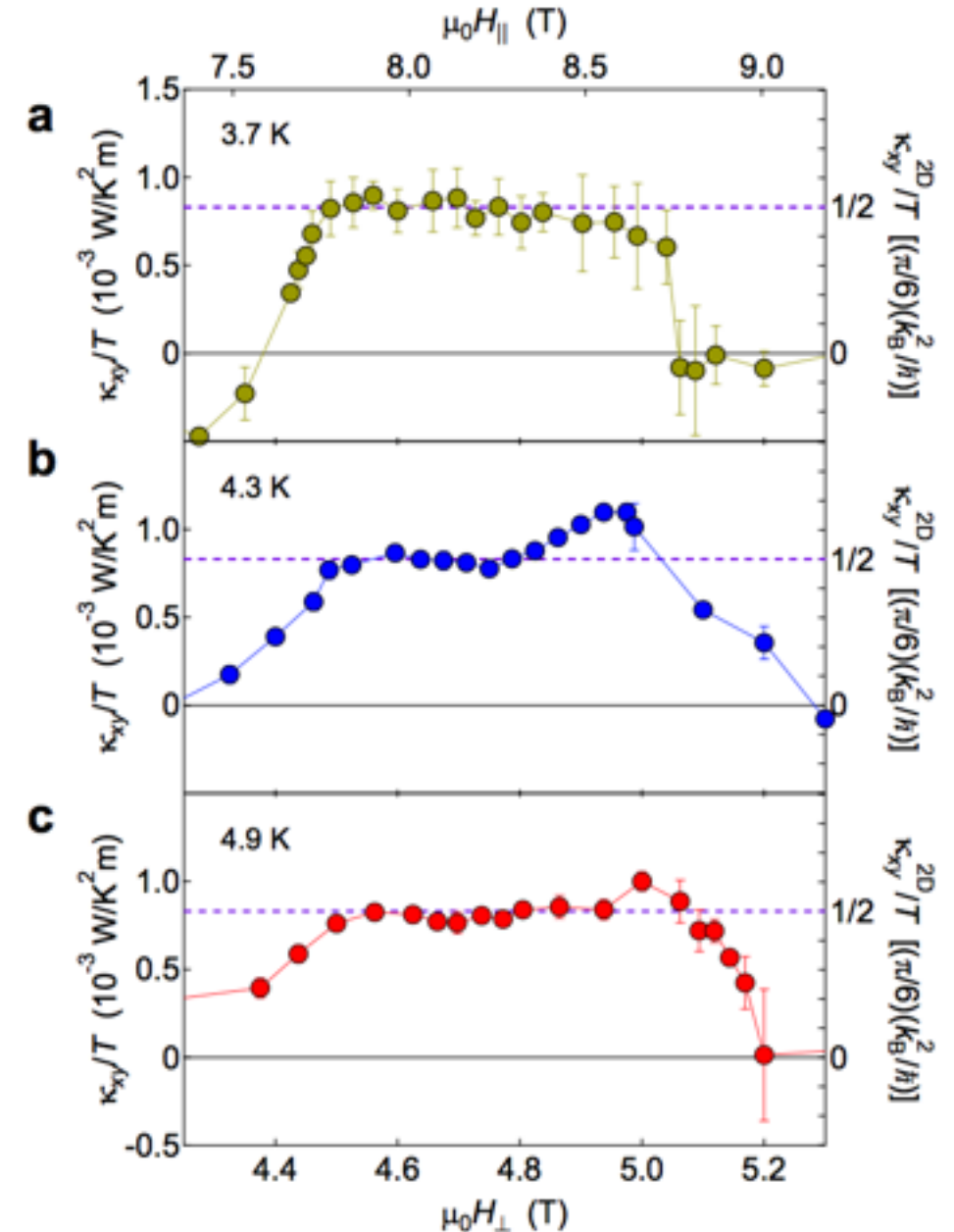
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Edge Majorana spinons?



How to tell a spinon from a spin wave?

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Bethe's solution: 1933
identification of spinons: 1981

What is the spin of a spin wave?

L. D. Faddeev and L. A. Takhtajan

Leningrad Branch of the Steklov Mathematical Institute, Leningrad, USSR

Received 15 July 1981. Available online 16 September 2002.

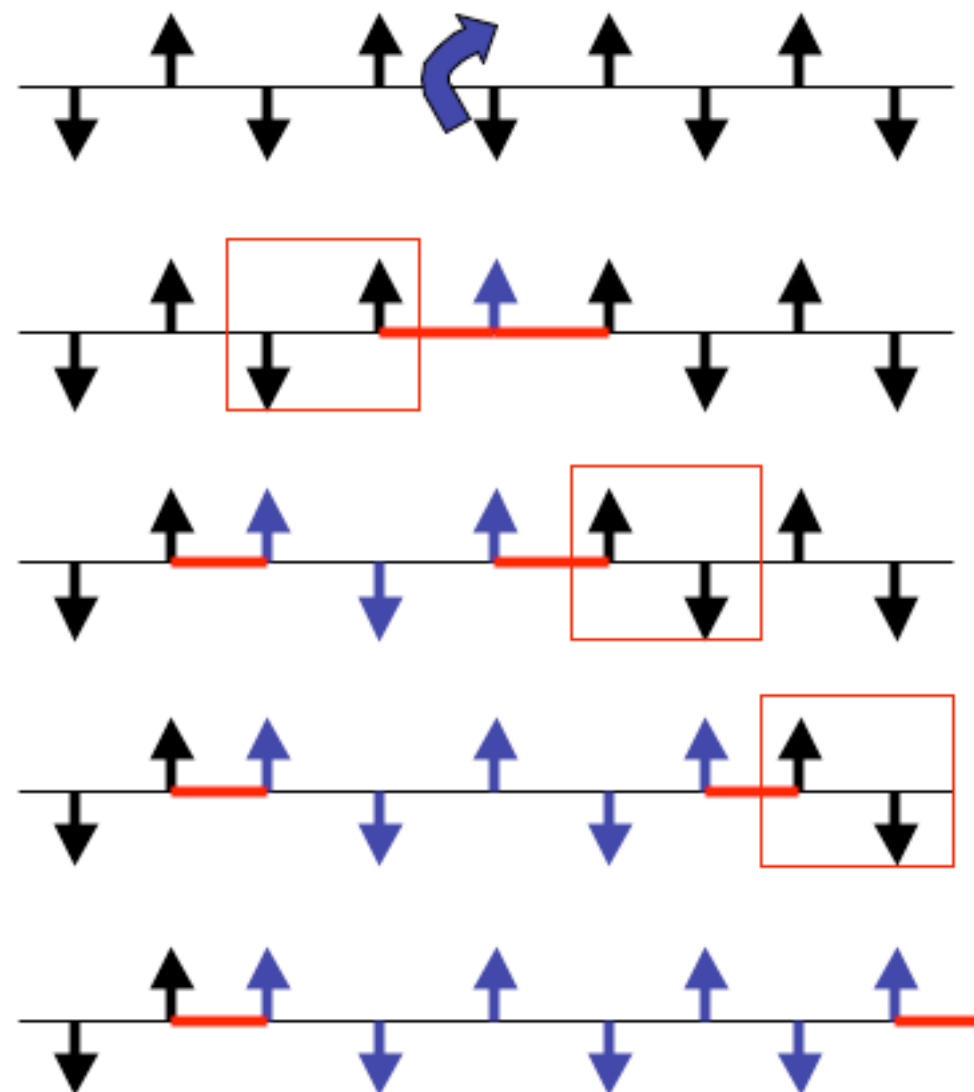
Abstract

We argue that the spin of a spin wave in the Heisenberg antiferromagnetic chain of spins $\frac{1}{2}$ is equal to $\frac{1}{2}$ rather than 1 as is generally considered to be true.

Article Outline

• References

S=1 spin wave
breaks into two
domain walls /
spinons:
hence each is
carrying S=1/2



Fully
understood:
d=1 chain

Two-spinon continuum of spin-1/2 chain

Spinon energy $\omega(k_x) = \frac{\pi J}{2} |\sin(k_x)|$ de Cloizeaux-Pearson dispersion, 1962

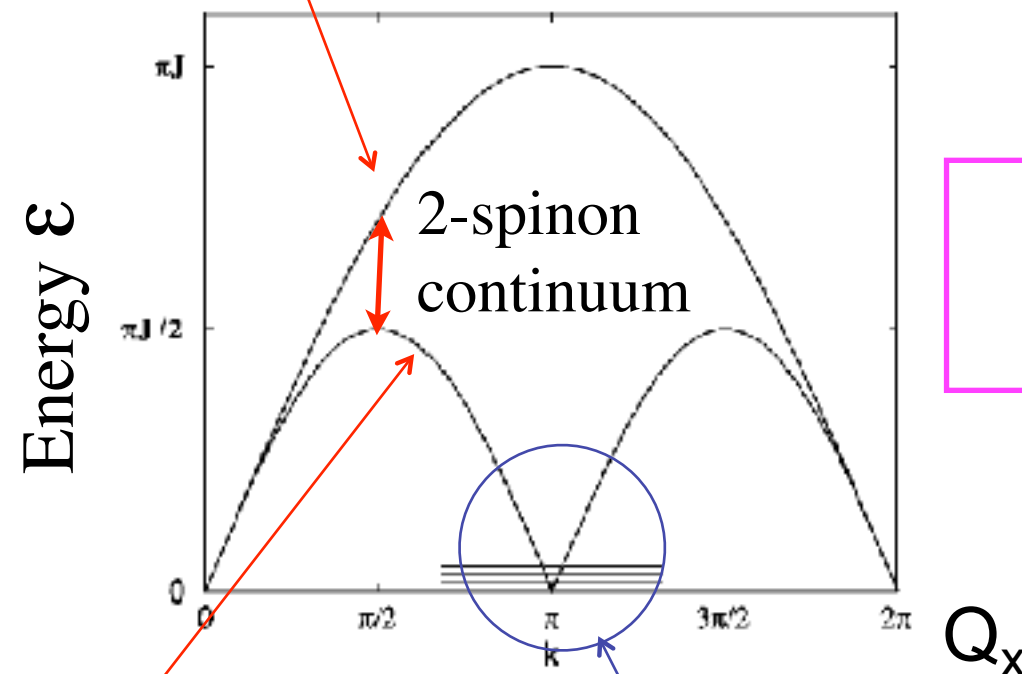
S=1 excitation $\begin{cases} \varepsilon = \omega(k_{x1}) + \omega(k_{x2}) \\ Q_x = k_{x1} + k_{x2} \end{cases}$

$$\varepsilon = \pi J \sin\left(\frac{Q_x}{2}\right) \cos\left(\frac{k_{x1} - k_{x2}}{2}\right)$$

Variables:
 k_{x1} and k_{x2}
or
 ε and Q_x

Upper boundary

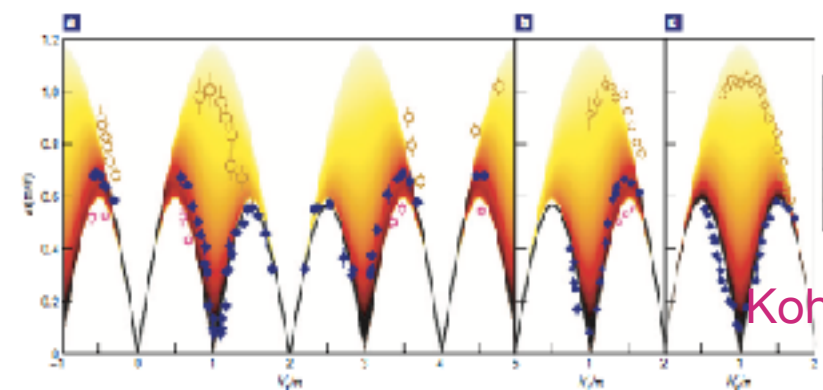
$$\omega_{2,\text{upper}} = \pi J \sin(Q_x/2) \\ k_{x1} = k_{x2} = Q_x/2$$



**Two particle
Excitation**

Lower
boundary

$$\omega_{2,\text{lower}} = \frac{\pi J}{2} \sin(Q_x) \\ k_{x1} = 0, k_{x2} = Q_x$$



Cs₂CuCl₄

Kohno, OS, Balents

Spinon

Magnetic

Resonance

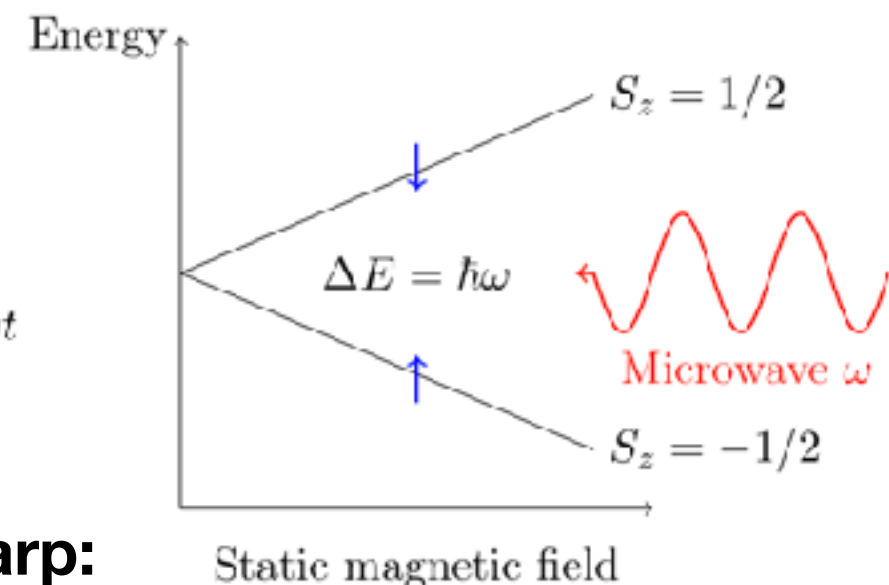
Electron Spin Resonance (ESR)

ESR measures absorption of electromagnetic radiation by a sample that is (typically) subjected to an external static magnetic field.

Linear response theory:

$$I(\vec{q} = 0, \omega) = \frac{1}{2} |h|^2 \omega \operatorname{Im} \chi_{\alpha\beta}(\vec{q} = 0, \omega)$$

$$\chi_{\alpha\beta}(\vec{q} = 0, \omega) = i \int_0^\infty dt \langle [S^\alpha(t), S^\beta(0)] \rangle e^{i\omega t}$$



For SU(2) invariant systems, completely sharp:

$$I(\omega) = \frac{1}{2} |h|^2 \omega \delta(\omega - B)$$

No matter how exotic the ground state is!

M. Oshikawa and I. Affleck, Phys. Rev. B 65, 134410 (2002).

The key point

- Perturbations violating $SU(2)$ symmetry do show up in ESR: **line shift** and **line width**!
- turn annoying material imperfection (spin-orbit, Dzyaloshinskii-Moriya) into a probe of exotic spin state and its excitations
- probe small-**q** excitations by ESR

absence of $SU(2)$ is actually not a restriction!

Condensed matter physics in 21 century: the age of **spin-orbit**

- ✓ spintronics
- ✓ topological insulators, Majorana fermions
- ✓ Kitaev's non-abelian honeycomb spin liquid

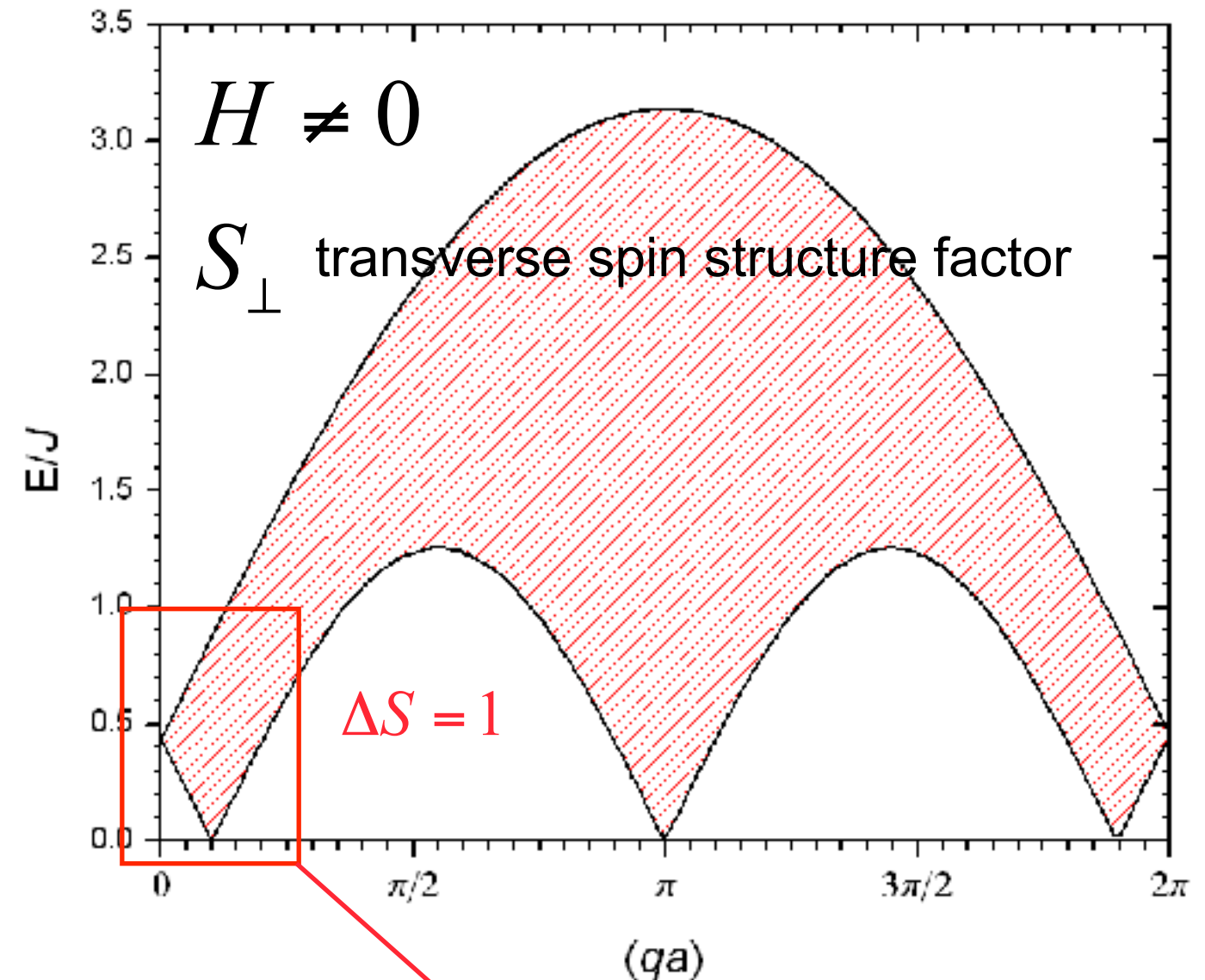
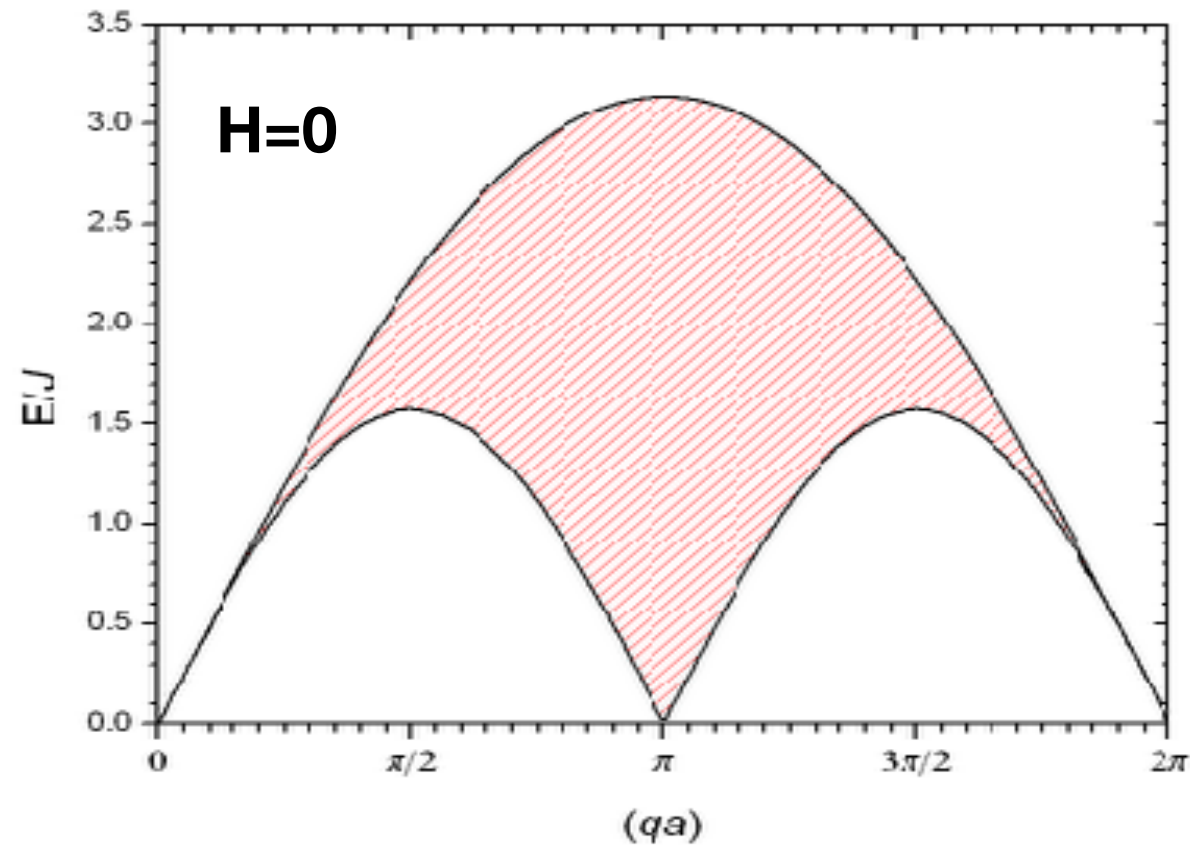
It is all about spin-orbit

Outline

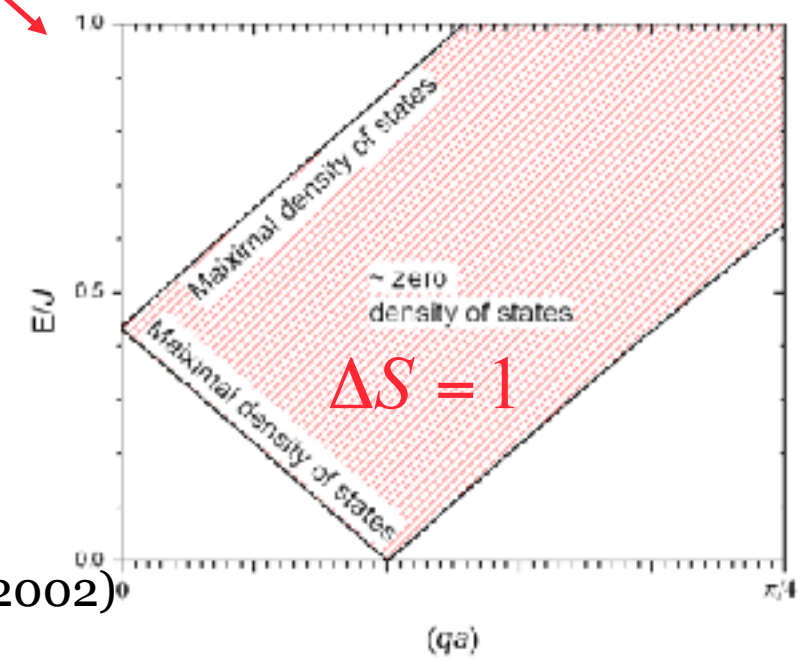
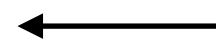
- *Main ingredients*
 - spin liquid
 - absence of spin-rotational symmetry (spin-orbit, DM, anisotropy...)
- *Line shape: ESR of one-dimensional spinon continuum in Cs_2CuCl_4*
- *Line shape: ESR of two-dimensional spinon continuum YbMgGaO_4*
- *Line width: ESR of spinons coupled to gauge field*
- *Conclusions*



Probing spinon continuum in one dimension



$$I(\omega) \propto \delta(\omega - H)$$



Dender et al, PRL 1997

Oshikawa, Affleck, PRB **65** 134410 (2002)

Uniform Dzyaloshinskii-Moriya interaction

$$H = \sum_{x,y,z} \boxed{JS_{x,y,z} \cdot S_{x+1,y,z}} - \boxed{D_{y,z} \cdot S_{x,y,z} \times S_{x+1,y,z}} - g\mu_B H \cdot S_{x,y,z}$$

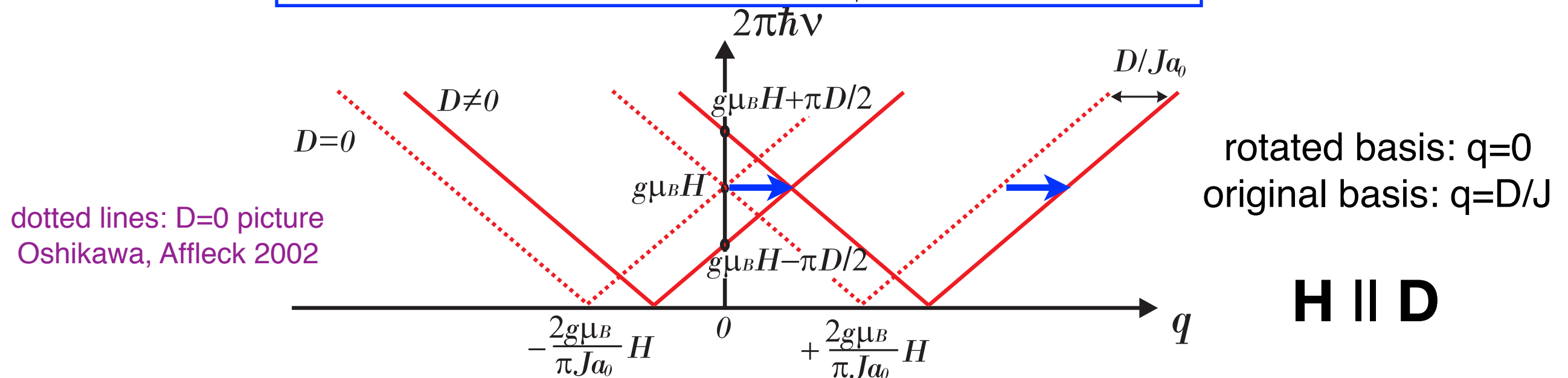
chain uniform DM along the chain magnetic field

!

Unitary rotation about **z**-axis $S^+(x) \rightarrow S^+(x)e^{i(D/J)x}, S^z(x) \rightarrow S^z(x)$

- removes DM term from the Hamiltonian (to D^2 accuracy)
- **boosts** momentum to $D/(Ja_0)$

$$q = 0 \rightarrow q = D/(Ja_0) \Rightarrow 2\pi\hbar v_{R/L} = g\mu_B H \pm \pi D/2$$



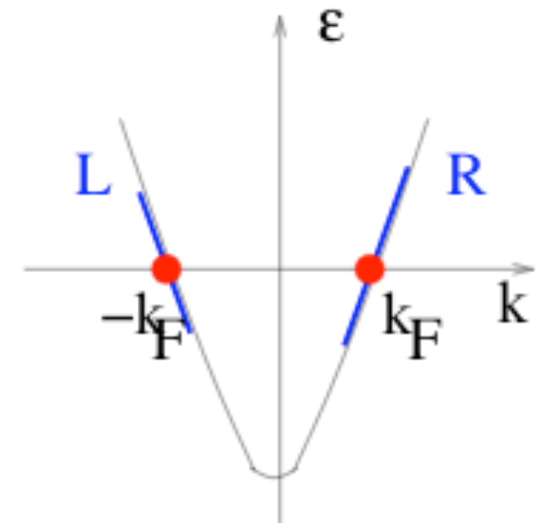
DM interaction allows to probe spinon continuum at finite “boost” momentum

Explanation II: arbitrary orientation

Relevant spin degrees of freedom

- Spin-1/2 AFM chain = half-filled (1 electron per site, $k_F = \pi/2a$) fermion chain

$$H_{\text{dirac}} = i v \int dx \sum_{s=\uparrow,\downarrow} (\Psi_{L,s}^\dagger \partial_x \Psi_{L,s} - \Psi_{R,s}^\dagger \partial_x \Psi_{R,s})$$



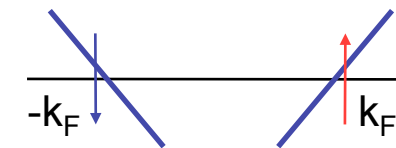
- $q=0$ fluctuations: right (R) and left (L) spin currents

$$\vec{M}_{R/L} = \Psi_{R/L,s}^\dagger \frac{\vec{\sigma}_{ss'}}{2} \Psi_{R/L,s'}$$

- $2k_F (= \pi/a)$ fluctuations: **charge** density wave ε , **spin** density wave N

Staggered
Magnetization $\mathbf{N}$

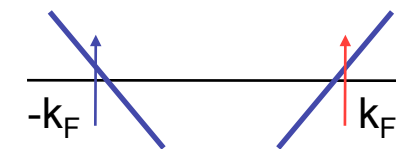
$$\begin{cases} N^+ \sim \Psi_{R\uparrow}^\dagger \Psi_{L\downarrow} + \text{h.c.} \\ N^z \sim \Psi_{R\uparrow}^\dagger \Psi_{L\uparrow} - \Psi_{R\downarrow}^\dagger \Psi_{L\downarrow} + \text{h.c.} \end{cases} \quad \text{Spin flip } \Delta S=1$$



Staggered
Dimerization

$$\varepsilon = (-1)^x S_x S_{x+a}$$

$$\varepsilon \sim i (\Psi_{R\uparrow}^\dagger \Psi_{L\uparrow} + \Psi_{R\downarrow}^\dagger \Psi_{L\downarrow} - \text{h.c.}) \quad \Delta S=0$$



Susceptibility

$$1/q \quad \chi_{1d}(q)$$

$$1/q$$

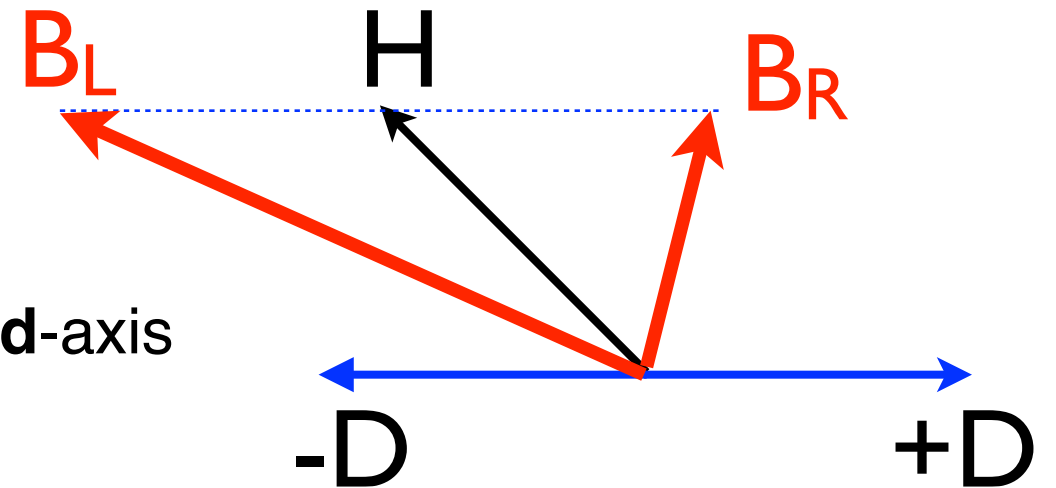
$$1/q$$

- Must be careful: **often** spin-charge separation must be enforced by hand

Arbitrary orientation of **H** and **D**

$$\mathbf{H} = \underbrace{\frac{2\pi\nu}{3}[(\vec{M}_R)^2 + (\vec{M}_L)^2]}_{\text{unperturbed chain}} \underbrace{-\frac{\nu D}{J}[M_R^d - M_L^d]}_{\text{uniform DM along the chain}} - g\mu_B H [M_R^z + M_L^z] \quad \text{magnetic field}$$

Uniform DM produces internal momentum-dependent magnetic field along **d**-axis



- Total field acting on right/left movers $g\mu_B \vec{H} \pm \hbar\nu \vec{D}/J$
- Hence ESR signals at $2\pi\hbar\nu_{R/L} = |g\mu_B \vec{H} \pm \hbar\nu \vec{D}/J|$
- Polarization: for $H=0$ maximal absorption when microwave field $\mathbf{h}_{\text{mw}}$ is perpendicular to the internal (DM) one. Hence $\mathbf{h}_{\text{mw}} \parallel \mathbf{b}$ is most effective.

Povarov et al, PRL 2011

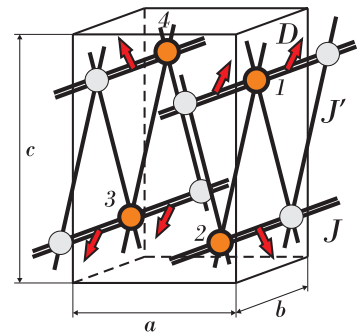
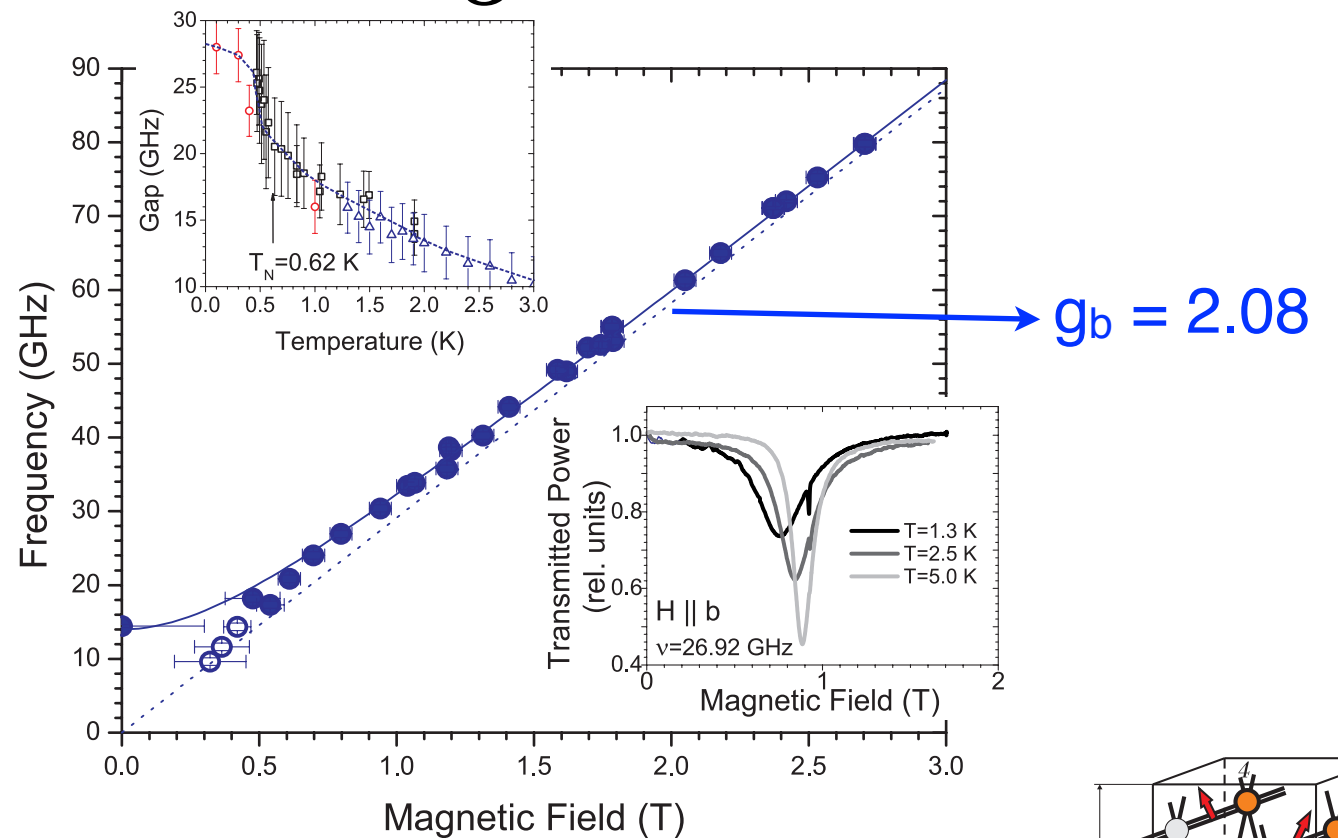
Gangadharaiah, Sun, OS, PRB 2008

- H along b-axis

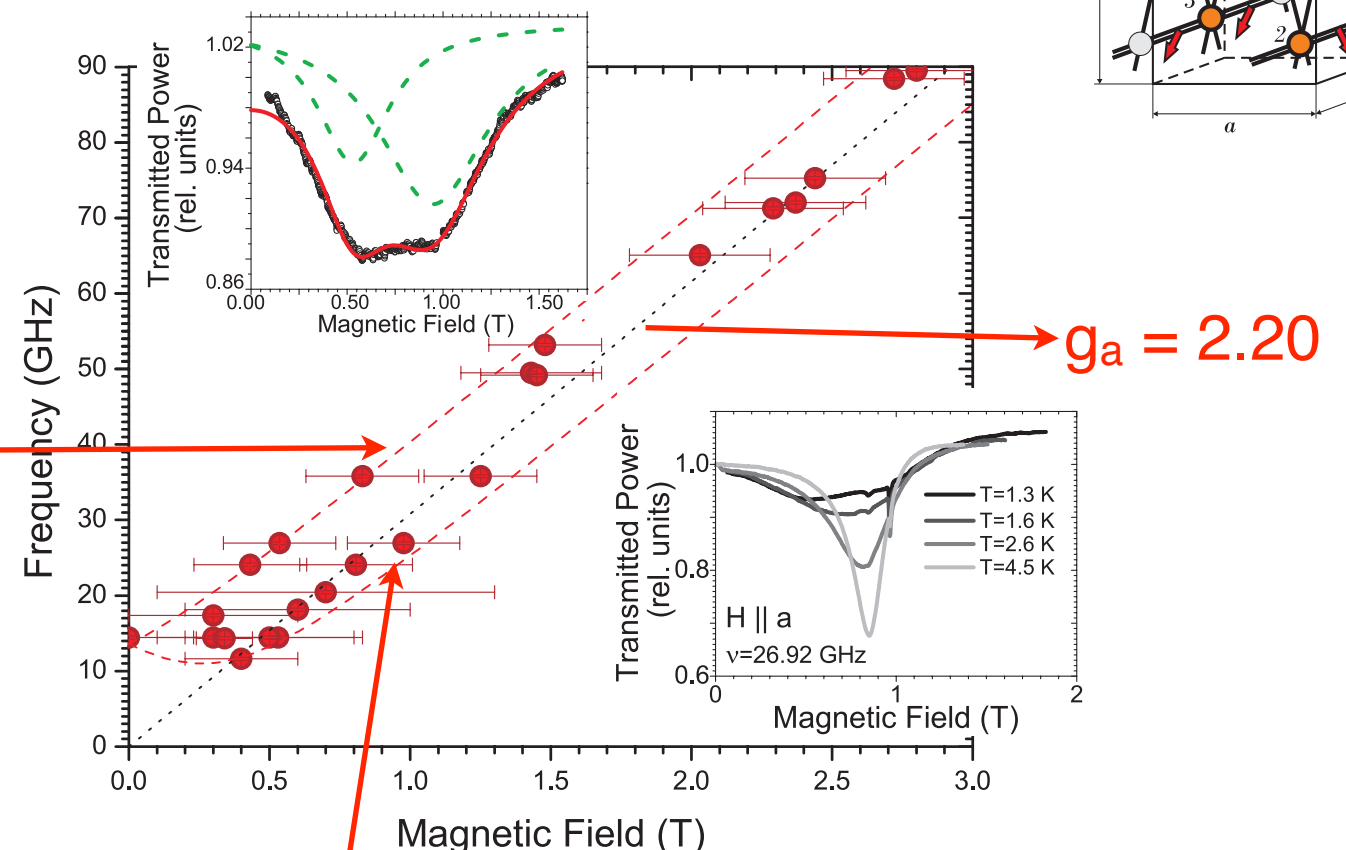
✓ gap-like behavior for $\nu > 17$ GHz

$$2\pi\hbar\nu = \sqrt{(g_b\mu_B H)^2 + \Delta^2}$$

✓ loss of intensity for $\nu < 17$ GHz



- H along a-axis:
splitting of the ESR line



Cs₂CuCl₄ ESR data

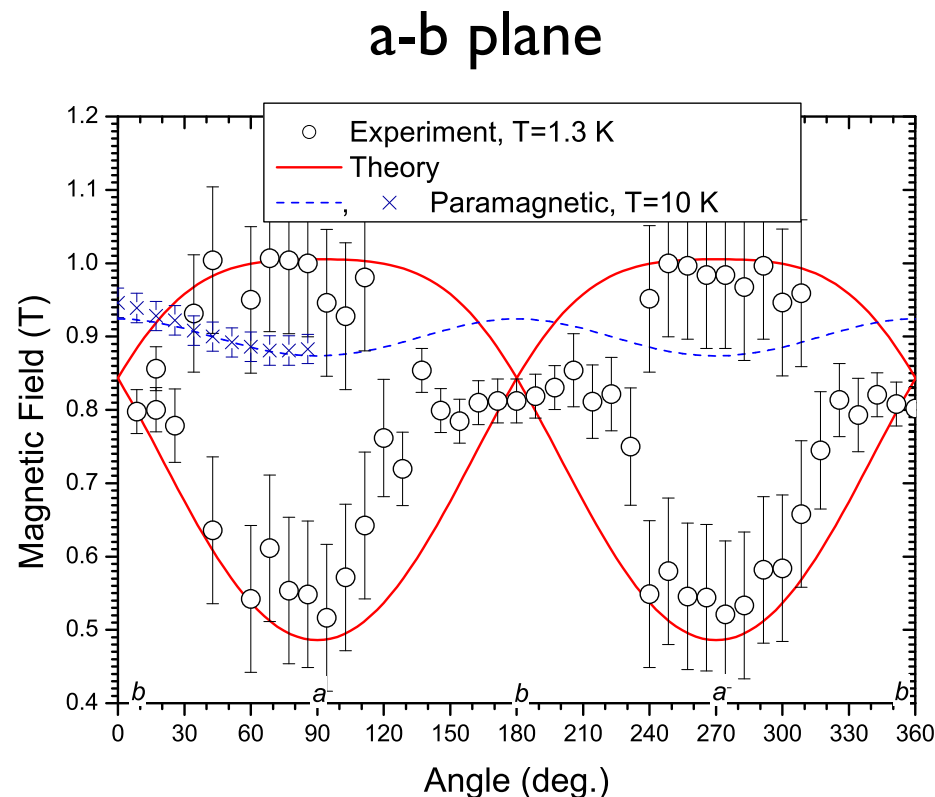
Modes of Magnetic Resonance in the Spin-Liquid Phase of Cs₂CuCl₄

K. Yu. Povarov,^{1,*} A. I. Smirnov,^{1,2} O. A. Starykh,^{3,4} S. V. Petrov,¹ and A. Ya. Shapiro⁵

- General orientation of **H** and **D**
- 4 sites/chains in unit cell

$$(2\pi\hbar\nu_R)^2 = (g_b\mu_B H_b)^2 + [g_a\mu_B H_a + (-1)^z \pi D_a/2]^2 + [g_c\mu_B H_c + (-1)^y \pi D_c/2]^2,$$

$$(2\pi\hbar\nu_L)^2 = (g_b\mu_B H_b)^2 + [g_a\mu_B H_a - (-1)^z \pi D_a/2]^2 + [g_c\mu_B H_c - (-1)^y \pi D_c/2]^2.$$



$$D_a/(4\hbar) = 8 \text{ GHz}$$

$$D_c/(4\hbar) = 11 \text{ GHz}$$

0.3 Tesla
0.4 Tesla

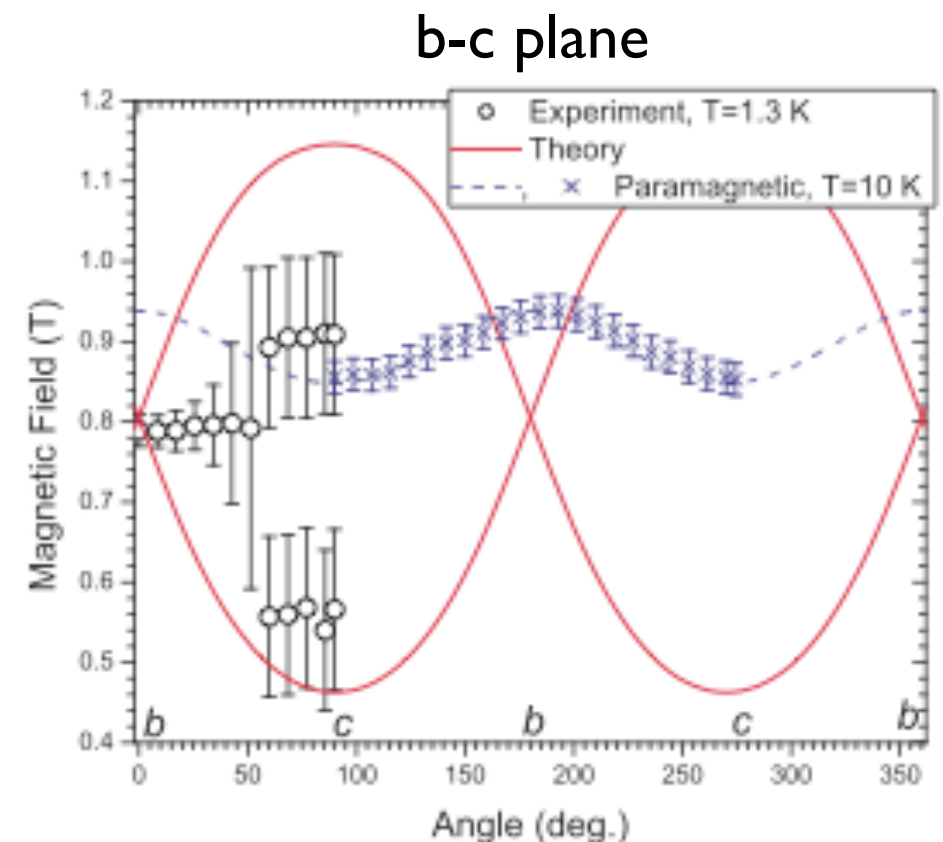
$$D \sim J/10$$

- for **H** along b-axis only: the “gap” is determined by the DM interaction strength

$$\Delta = \frac{\pi}{2} \sqrt{D_a^2 + D_c^2} \rightarrow (2\pi\hbar) 13.6 \text{ GHz}$$

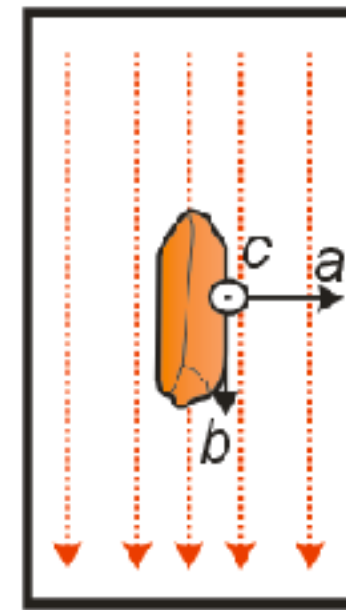
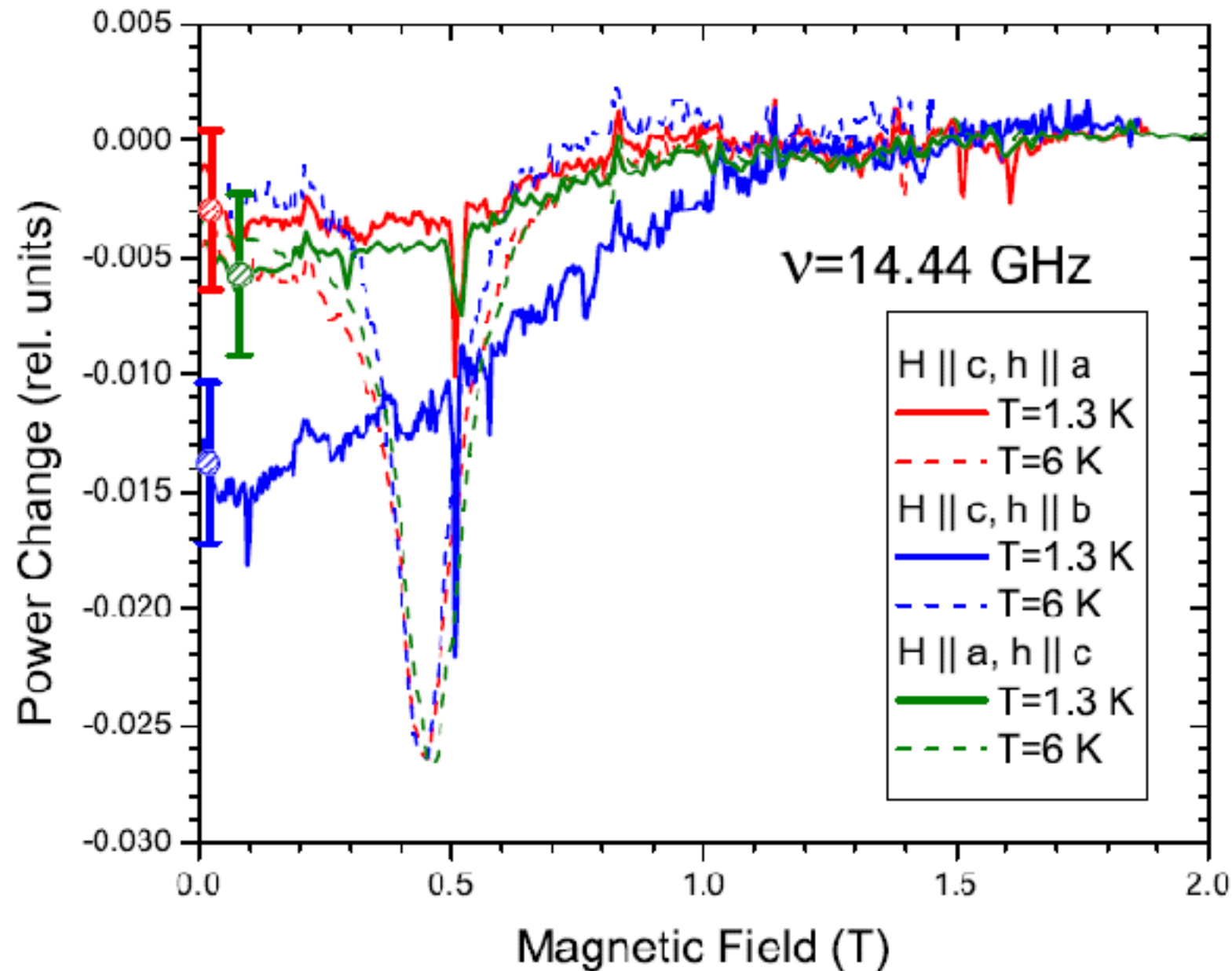
Linear in **T** line width

S. C. Furuya Phys. Rev. B 95, 014416 (2017)

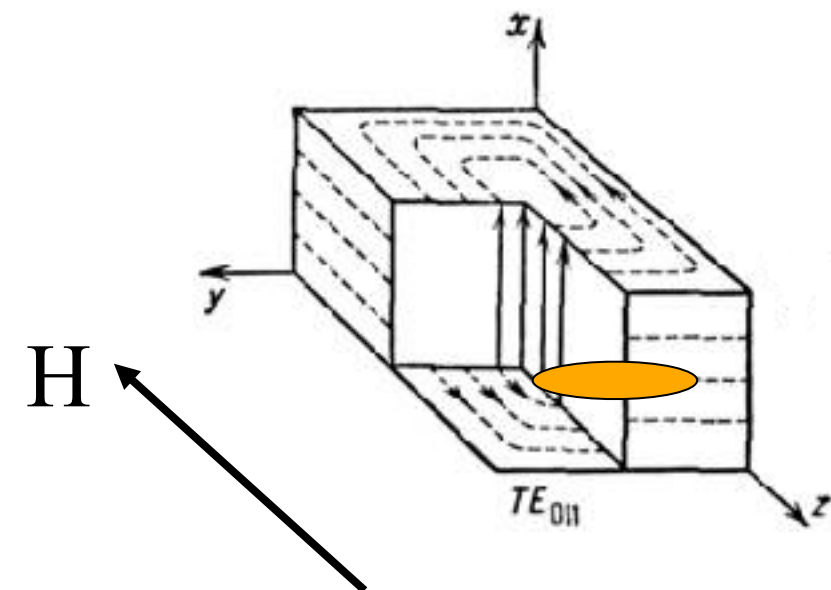
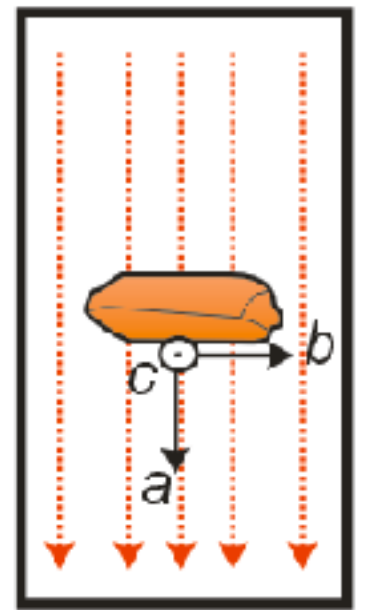


Zero-field absorption:

- ✓ 1) ESR absorption in the absence of H $\nu \sim \sqrt{D_a^2 + D_c^2}$
- ✓ 2) strong polarization dependence in zero field



h



The largest absorption occurs when microwave field $h(t)$ is lined along crystal b-axis, $h \parallel b$ [so that it is perpendicular to the $\mathbf{D}$ vector in a - c plane] Povarov et al, PRL 2011

Electron spin resonance in a model $S = \frac{1}{2}$ chain antiferromagnet with a uniform Dzyaloshinskii-Moriya interaction

A. I. Smirnov,¹ T. A. Soldatov,^{1,2} K. Yu. Povarov,^{3,*} M. Hälg,³ W. E. A. Lorenz,³ and A. Zheludev³

¹*P. L. Kapitza Institute for Physical Problems, RAS, 119334 Moscow, Russia*

²*Moscow Institute for Physics and Technology, 141700 Dolgoprudny, Russia*

³*Neutron Scattering and Magnetism, Laboratory for Solid State Physics, ETH Zürich, Switzerland*

(Received 27 July 2015; revised manuscript received 6 October 2015; published 22 October 2015)

The electron spin resonance spectrum of a quasi-1D $S = 1/2$ antiferromagnet $\text{K}_2\text{CuSO}_4\text{Br}_2$ was found to demonstrate an energy gap and a doublet of resonance lines in a wide temperature range between the Curie-Weiss and Néel temperatures. This type of magnetic resonance absorption corresponds well to the two-spinon continuum of excitations in $S = 1/2$ antiferromagnetic spin chain with a uniform Dzyaloshinskii-Moriya interaction between the magnetic ions. A resonance mode of paramagnetic defects demonstrating strongly anisotropic behavior due to interaction with spinon excitations in the main matrix is also observed.

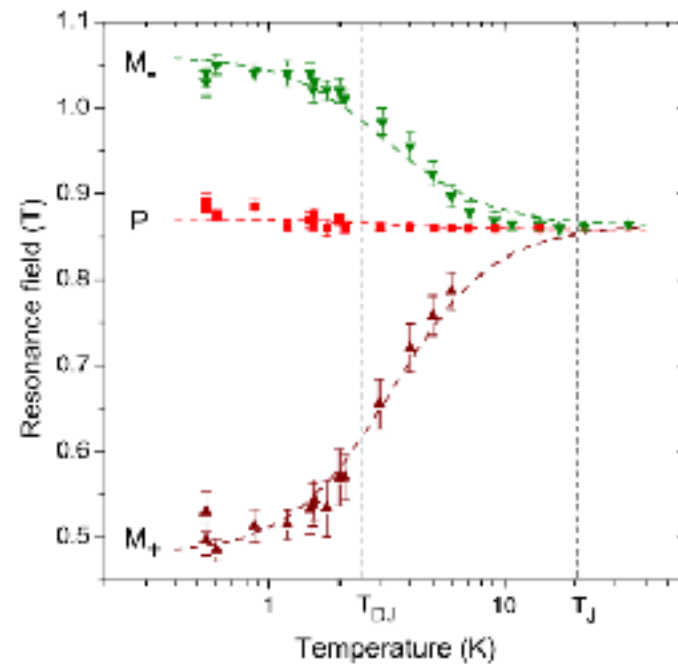


FIG. 4. (Color online) Temperature dependence of 27.83 GHz ESR fields at $\mathbf{H} \parallel b$ for the components of the doublet M_+ , M_- and of the paramagnetic line P , associated with the defects. Characteristic temperatures marked on the horizontal axis are $T_{DJ} = \sqrt{DJ}/k_B$ and $T_N = J/k_B$. Dashed lines are guide to the eyes.

$$J = 20 \text{ K}, D = 0.27 \text{ K}$$

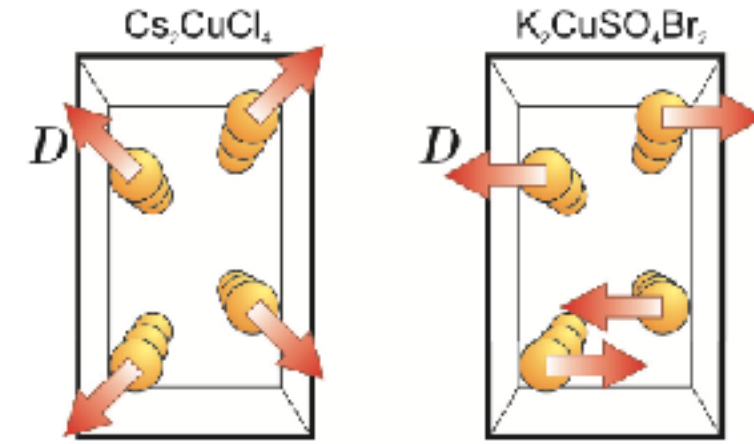


FIG. 2. (Color online) Comparison of the Dzyaloshinskii-Moriya vectors in Cs_2CuCl_4 and $\text{K}_2\text{CuSO}_4\text{Br}_2$. Perspective view along the spin chains.

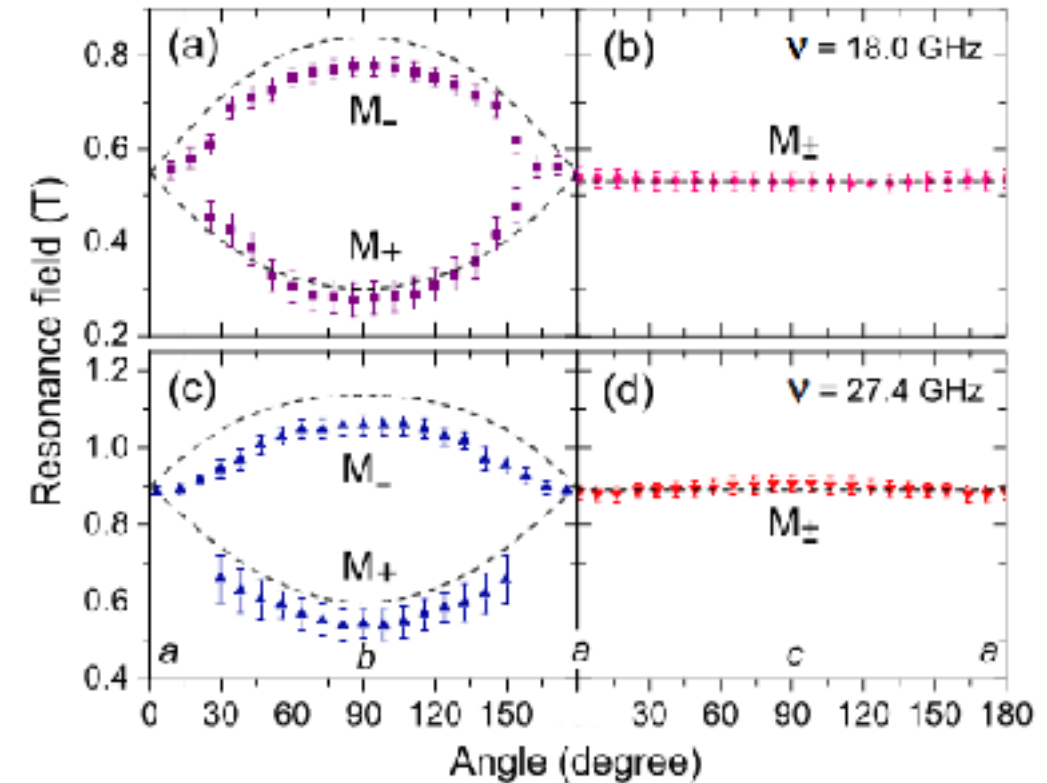
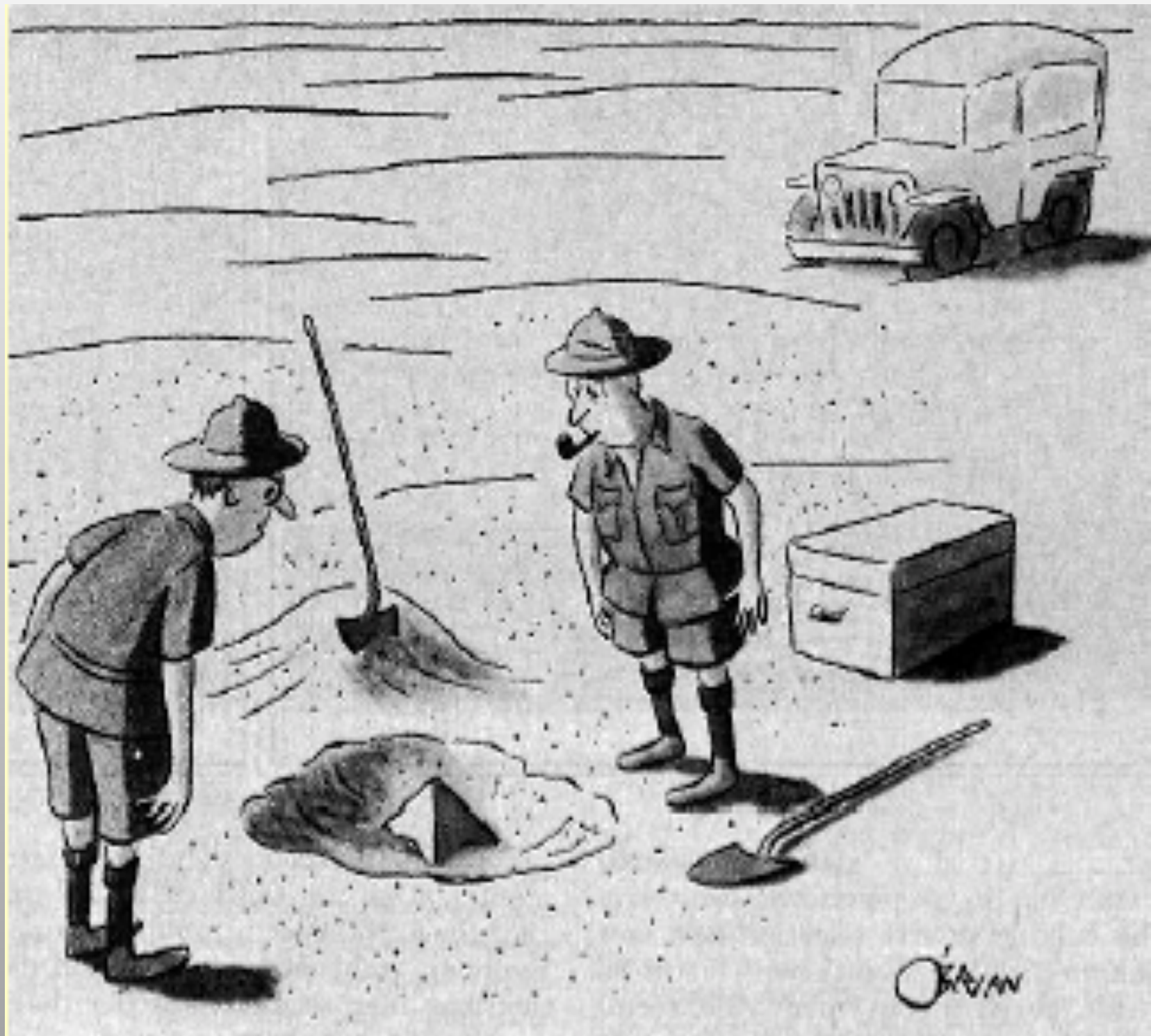


FIG. 8. (Color online) Angular dependencies of the resonant fields at two frequencies for $T = 1.3 \text{ K}$. 18.0 GHz ESR fields for field in ab plane (upper left panel), and in ac plane (upper right panel). 27.4 GHz ESR fields for field in ab plane (lower left panel), and in ac plane (lower right panel). Dashed lines are theoretical dependencies according to (2).

Two-dimensional spin liquids with fractionalized excitations



“This could be the discovery of the century. Depending, of course, on how far down it goes”

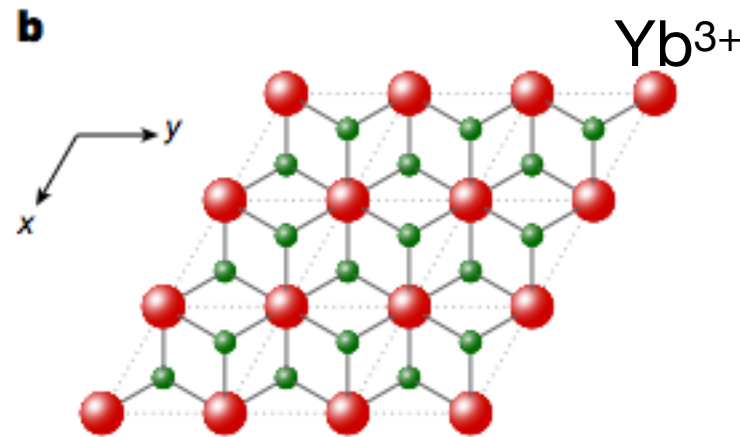
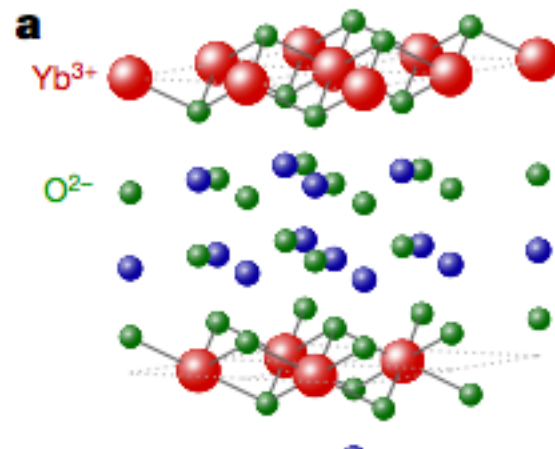
Two (and three) dimensional spin liquids

No magnetic order
Significant spin-orbit interaction
Fractionalized excitations

Kitaev materials
Pyrochlores
Iridates

YbMgGaO₄
interesting
promising
simple lattice

*Spinon Fermi surface
state suggested by
early neutron exps!*



Higher dimensional extension (weak Mott insulators)

- origin of DM: spin-orbit tunneling in Hubbard model

$$\hat{H} = \sum_{i,j} \{c_{i,\alpha}^\dagger (-t\delta_{\alpha\beta} + i\vec{\lambda}_{ij} \cdot \vec{s}_{\alpha\beta}) c_{j,\beta} + \text{H.c.}\} + U \sum_i n_{i\uparrow} n_{i\downarrow}.$$

- 2D square lattice with **uniform spin-orbit** interaction (YBa₂ Cu₃ O_{6+x})

$$\vec{\lambda}_{ij} = \lambda \hat{z} \times (\vec{r}_i - \vec{r}_j)$$

Coffey, Rice, Zhang 1991

Shekhtman, Entin-Wohlman, Aharony 1992

Bonesteel 1993

- (Lattice) spin-orbit interaction of Rashba type

$$\hat{H}_{\text{SO}}(\mathbf{k}) = -2\lambda \sum_k c_{k,\alpha}^\dagger \{ \hat{s}_x \sin[k_y] - \hat{s}_y \sin[k_x] \} c_{k,\beta}$$

- Transition to **spinons** via **slave-rotor** formulation $c_{r,\sigma} = f_{r,\sigma} e^{i\theta_r}$

Florens and Georges 2004

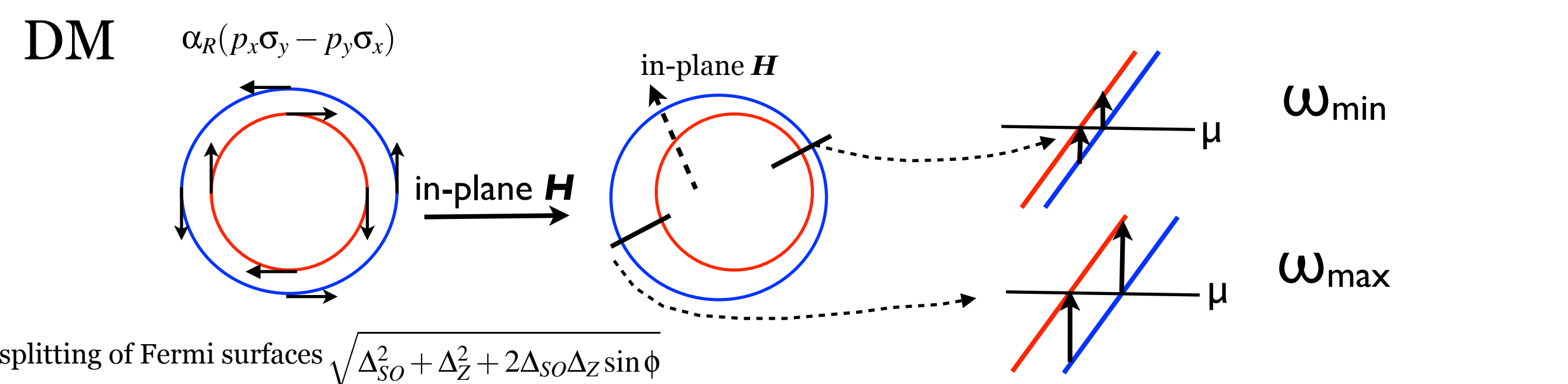
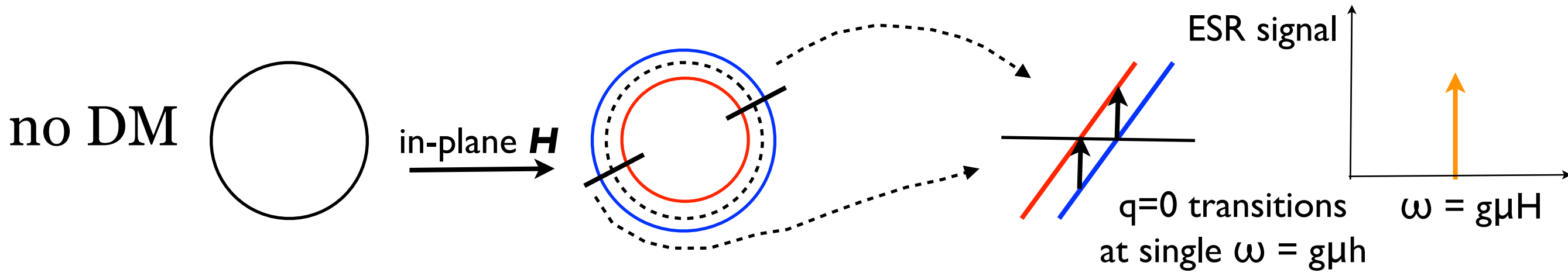
S.-S. Lee and P. A. Lee 2005

- (mean-field) Rashba Hamiltonian for free spinons ($f_{r,s}$)

$$\hat{H}_f = \sum_{i,j} f_{i,\alpha}^\dagger (-t\delta_{\alpha\beta} + i\vec{\lambda}_{ij}^{\text{eff}} \cdot \vec{s}_{\alpha\beta}) f_{j,\beta} - \vec{H} \cdot f_{i,\alpha}^\dagger \vec{s}_{\alpha\beta} f_{j,\beta}$$

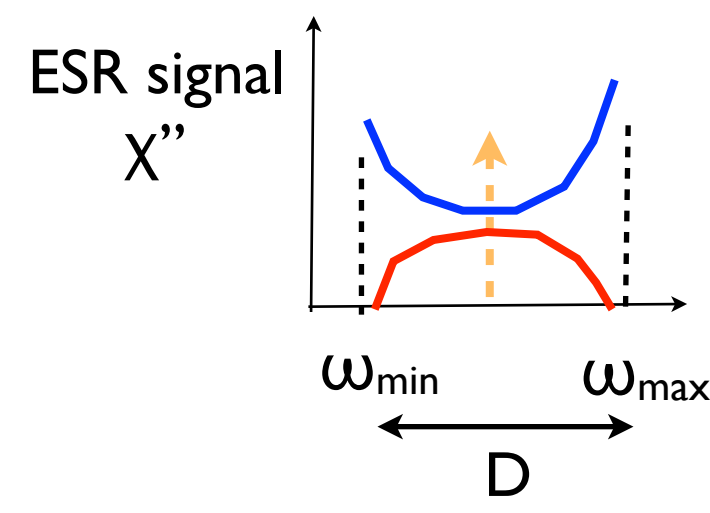
Glenn, OS, Raikh, PRB 2012

Estimates for 2d spinon gas using Rashba model as an example



Glenn, OS, Raikh 2012

Energy absorption due to microwave $h(t)$



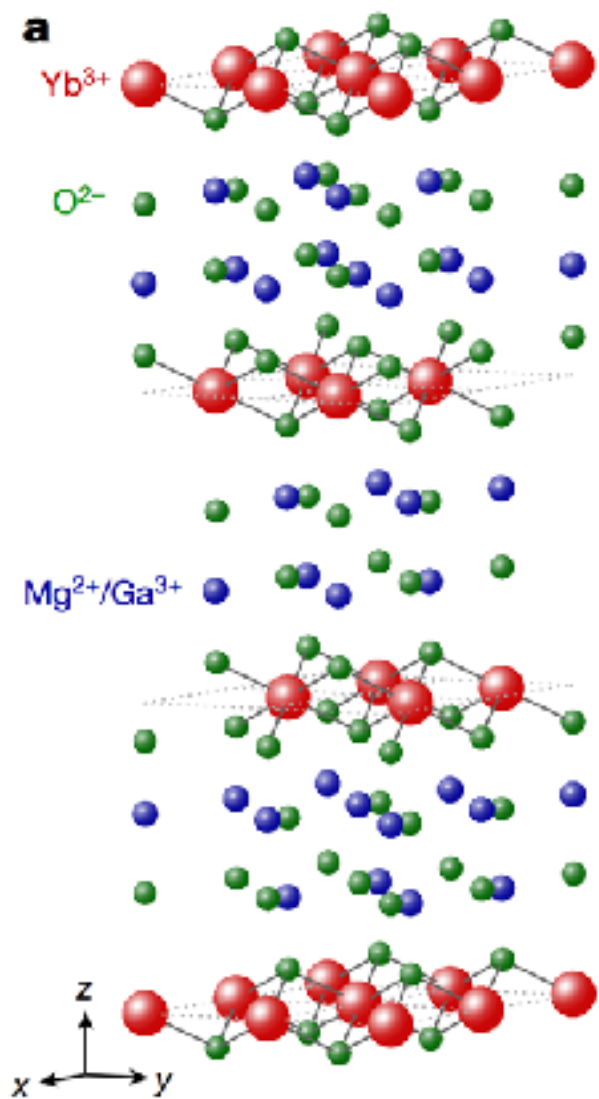
the width of the line $\sim \min(H, \alpha_R k_F)$
line shape is strongly polarization-dependent:
 $[\omega - \omega_{min/max}]^{-1/2}$ for $h(t)$ **perpendicular** to H
 $[\omega - \omega_{min/max}]^{1/2}$ for $h(t)$ **parallel** to H

2D spin liquid: YbMgGaO₄

Sci Rep. 2015 Nov 10;5:16419. doi: 10.1038/srep16419.

Gapless quantum spin liquid ground state in the two-dimensional spin-1/2 triangular antiferromagnet YbMgGaO₄.

Li Y¹, Liao H², Zhang Z³, Li S³, Jin F¹, Ling L⁴, Zhang L⁴, Zou Y⁴, Pi L⁴, Yang Z⁵, Wang J⁶, Wu Z⁷, Zhang Q^{1,8}.

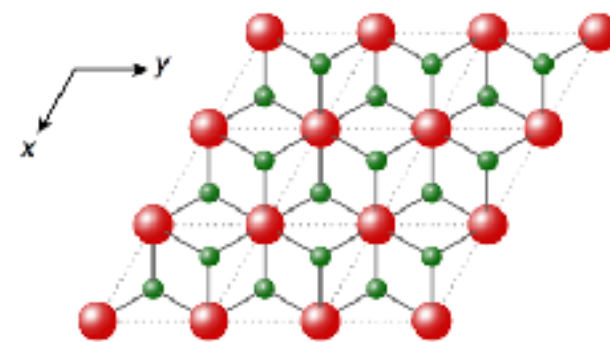


Rare-Earth Triangular Lattice Spin Liquid: A Single-Crystal Study of YbMgGaO₄

Yuesheng Li, Gang Chen, Wei Tong, Li Pi, Juanjuan Liu, Zhaorong Yang, Xiaoqun Wang, and Qingming Zhang
Phys. Rev. Lett. **115**, 167203 – Published 16 October 2015

Muon Spin Relaxation Evidence for the U(1) Quantum Spin-Liquid Ground State in the Triangular Antiferromagnet YbMgGaO₄

Yuesheng Li, Devashibhai Adroja, Pabitra K. Biswas, Peter J. Baker, Qian Zhang, Juanjuan Liu, Alexander A. Tsirlin, Philipp Gegenwart, and Qingming Zhang
Phys. Rev. Lett. **117**, 097201 – Published 23 August 2016



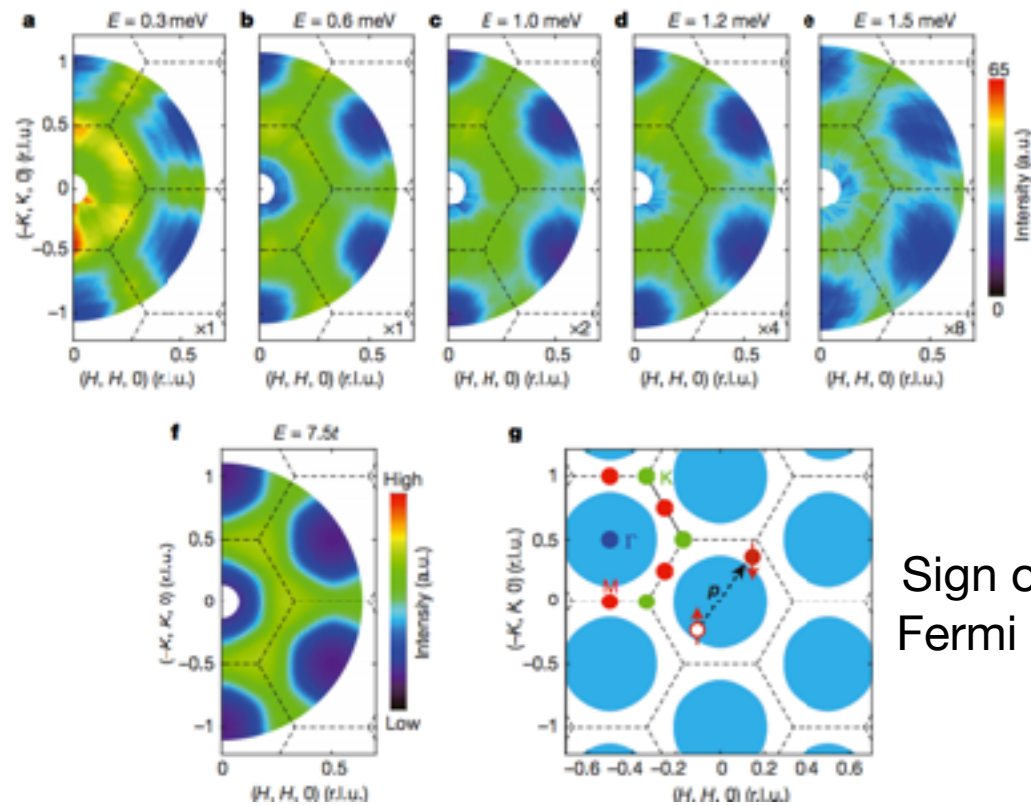
Strong spin-orbit coupling

$$\begin{aligned} \mathcal{H} = & \sum_{\langle i,j \rangle} [J_1^{zz} S_i^z S_j^z + J_1^{\pm} (S_i^+ S_j^- + S_i^- S_j^+)] \\ & + J_1^{\pm\pm} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-) \\ & - \frac{iJ_1^{z\pm}}{2} (\gamma_{ij}^* S_i^+ S_j^z - \gamma_{ij} S_i^- S_j^z + \langle i \leftrightarrow j \rangle)] \\ & + \sum_{\langle\langle i,j \rangle\rangle} [J_2^{zz} S_i^z S_j^z + J_2^{\pm} (S_i^+ S_j^- + S_i^- S_j^+)] \\ & - \mu_0 \mu_B \sum [g_{\perp} (H^x S_i^x + H^y S_i^y) + g_{\parallel} H^z S_i^z] \end{aligned}$$

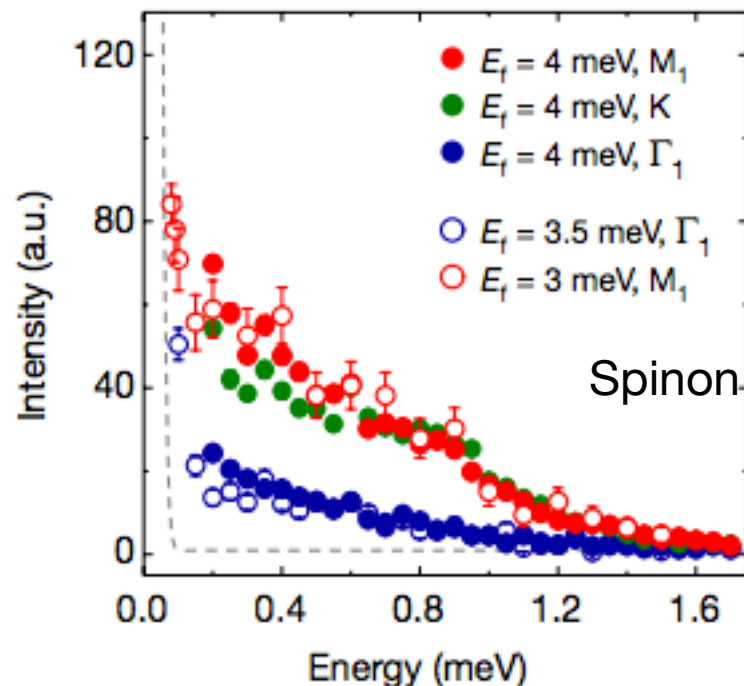
Figure from: Nature 540, pp 559–562 (2016).

Evidence for a spinon Fermi surface in a triangular-lattice quantum-spin-liquid candidate

Yao Shen¹, Yao-Dong Li², Hongliang Wu³, Yuesheng Li⁴, Shoudong Shen¹, Dingying Pan¹, Qisi Wang¹, H. C. Walker⁵, E. Steffen⁶, M. Boehn⁷, Yaping Hao⁸, D. L. Quintero-Castro⁹, L. W. Harriger¹⁰, M. D. Famlitzek¹¹, Tijie Han⁹, Siqin Yeag⁹, Qingming Zhang^{10,11}, Gang Chen^{11,12} & Jun Zhao^{1,13}



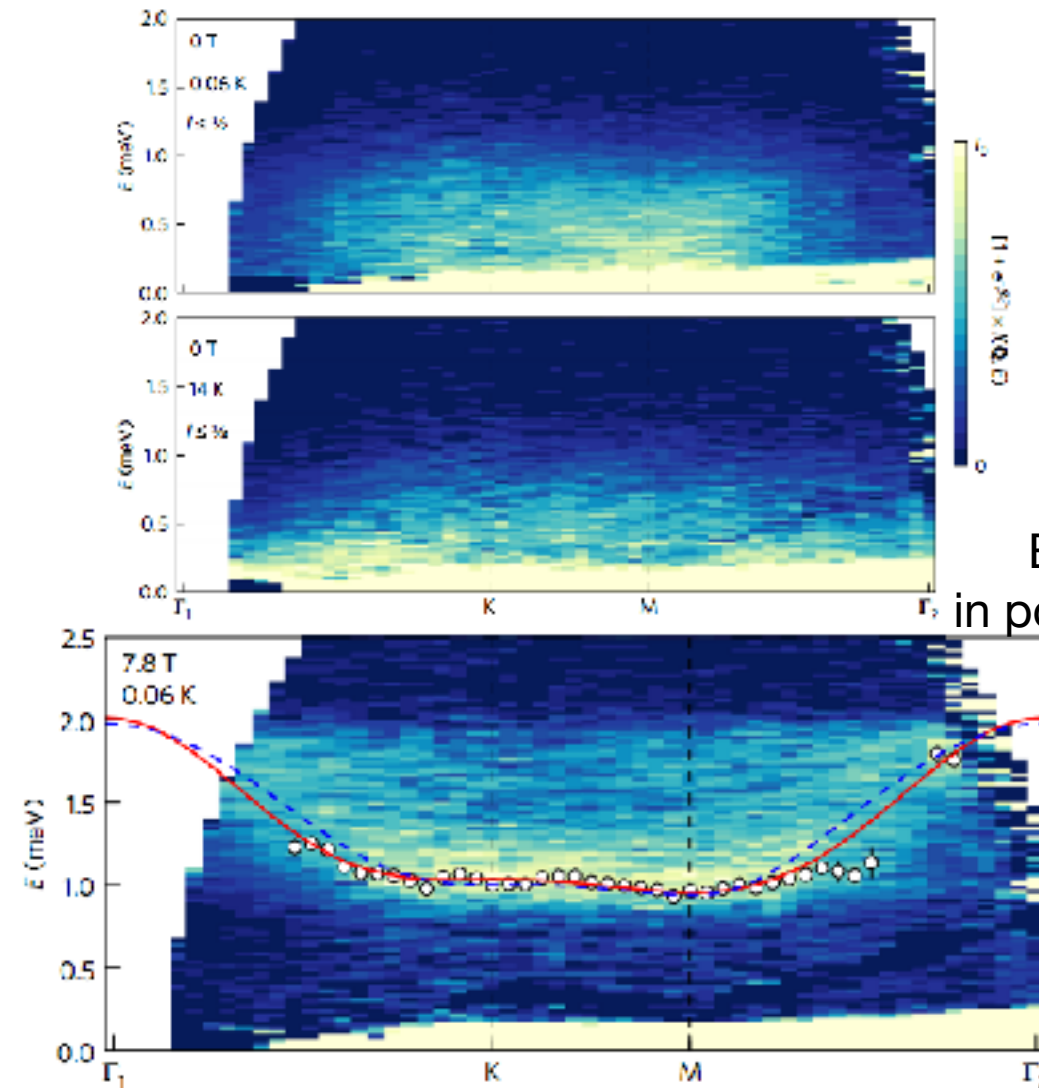
Sign of spinon Fermi surface?



Spinon continuum?

Continuous excitations of the triangular-lattice quantum spin liquid YbMgGaO₄

Joseph A. M. Paddison¹, Marcus Daum^{1†}, Zhiling Dun^{2†}, Georg Ehlers³, Yaohua Liu³, Matthew B. Stone³, Haidong Zhou³ and Martin Mourigal^{1*}



Broad signal in polarized phase?

very disordered

$$\begin{aligned} \mathcal{H} = & \sum_{\langle ij \rangle} \left[J_z^z S_i^z S_j^z + J_1^\pm (S_i^+ S_j^- + S_i^- S_j^+) + J_1^{\pm\pm} (\gamma_\theta S_i^+ S_j^+ + \gamma_\theta^* S_i^- S_j^-) \right] \\ & + \sum_{\langle ij \rangle} \left[J_2^z S_i^z S_j^z + J_2^\pm (S_i^+ S_j^- + S_i^- S_j^+) \right] \\ & - \mu_0 \mu_B \sum_i \left[g_\perp (H^x S_i^x + H^y S_i^y) + g_\parallel H^z S_i^z \right] \end{aligned} \quad (1)$$

Spinon Fermi surface $U(1)$ spin liquid in the spin-orbit-coupled triangular-lattice Mott insulator YbMgGaO_4

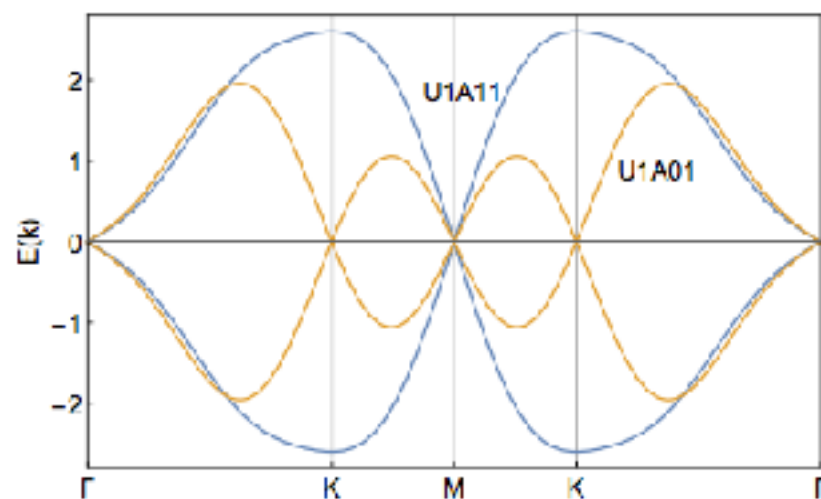
Yao-Dong Li,¹ Yuan-Ming Lu,² and Gang Chen^{1,3,*}

Spinon Magnetic Resonance of Quantum Spin Liquids

Zhu-Xi Luo,^{1,*} Ethan Lake,¹ Jia-Wei Mei,² and Oleg A. Starykh^{1,†}

Spinon mean-field Hamiltonian
derived with the help of
Projective Symmetry Group (PSG) analysis

$$S_{\mathbf{r}}^a = \frac{1}{2} f_{\mathbf{r}\alpha}^\dagger \sigma_{\alpha\beta}^a f_{\mathbf{r}\beta}$$

FIG. S-5. Spinon dispersions $E_{1,2}(\mathbf{k})$ along the line Γ -K-M-K- Γ for U1A11 and U1A01 states.

Basic idea: physical spin S is bilinear of spinons f ,
spinons have bigger symmetry group than spins,
this leads to gauge freedom and
different classes of possible mean-fields.
These classes describe the same spin problem.

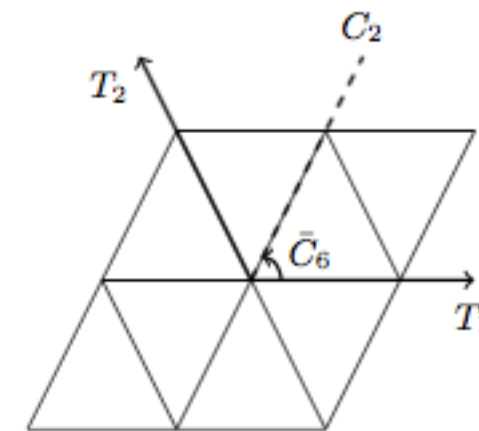
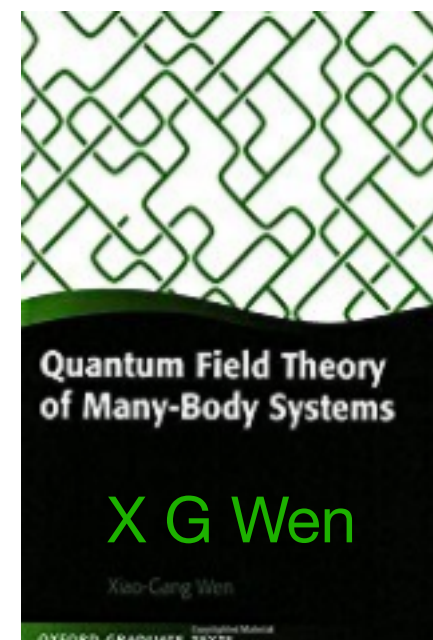


FIG. S-1. The symmetry operations.

Dirac spectrum!



Mean-field Hamiltonians

Eight types:

U1A00, U1A01, U1A10, U1A11; U1Bxx
 SU(2) trivial **Our focus** π -flux

Accidental Symmetry
 = ideal ESR

Symmetry	Transformation Rules
T_1	$\begin{cases} f_{(x,y)\uparrow} \rightarrow f_{(x+1,y)\uparrow} \\ f_{(x,y)\downarrow} \rightarrow f_{(x+1,y)\downarrow} \end{cases}$
T_2	$\begin{cases} f_{(x,y)\uparrow} \rightarrow f_{(x,y+1)\uparrow} \\ f_{(x,y)\downarrow} \rightarrow f_{(x,y+1)\downarrow} \end{cases}$
C_2	$\begin{cases} f_{(x,y)\uparrow} \rightarrow e^{i\pi/6} f_{(y,x)\uparrow}^\dagger \\ f_{(x,y)\downarrow} \rightarrow e^{-i\pi/6} f_{(y,x)\downarrow}^\dagger \end{cases}$
$\bar{C}_6$	$\begin{cases} f_{(x,y)\uparrow} \rightarrow e^{i\pi/3} f_{(x-y,x)\downarrow}^\dagger \\ f_{(x,y)\downarrow} \rightarrow -e^{-i\pi/3} f_{(x-y,x)\uparrow}^\dagger \end{cases}$
$\mathcal{T}$	$\begin{cases} f_{(x,y)\uparrow} \rightarrow f_{(x,y)\downarrow} \\ f_{(x,y)\downarrow} \rightarrow -f_{(x,y)\uparrow} \end{cases}$

TABLE I. U1A11 PSG analysis.

Symmetry	Transformation Rules
T_1	$\begin{cases} f_{(x,y)\uparrow} \rightarrow f_{(x+1,y)\uparrow} \\ f_{(x,y)\downarrow} \rightarrow f_{(x+1,y)\downarrow} \end{cases}$
T_2	$\begin{cases} f_{(x,y)\uparrow} \rightarrow f_{(x,y+1)\uparrow} \\ f_{(x,y)\downarrow} \rightarrow f_{(x,y+1)\downarrow} \end{cases}$
C_2	$\begin{cases} f_{(x,y)\uparrow} \rightarrow -e^{i\pi/6} f_{(y,x)\downarrow}^\dagger \\ f_{(x,y)\downarrow} \rightarrow e^{-i\pi/6} f_{(y,x)\uparrow}^\dagger \end{cases}$
$\bar{C}_6$	$\begin{cases} f_{(x,y)\uparrow} \rightarrow e^{i\pi/3} f_{(x-y,x)\downarrow}^\dagger \\ f_{(x,y)\downarrow} \rightarrow -e^{-i\pi/3} f_{(x-y,x)\uparrow}^\dagger \end{cases}$
$\mathcal{T}$	$\begin{cases} f_{(x,y)\uparrow} \rightarrow f_{(x,y)\downarrow} \\ f_{(x,y)\downarrow} \rightarrow -f_{(x,y)\uparrow} \end{cases}$

TABLE II. U1A01 PSG analysis.

$$\begin{aligned}
 H = \sum_{x,y} \{ & t_1 [i f_{(x+1,y)\uparrow}^\dagger f_{(x,y)\uparrow} + i f_{(x,y+1)\uparrow}^\dagger f_{(x,y)\uparrow} - i f_{(x+1,y+1)\uparrow}^\dagger f_{(x,y)\uparrow} \\
 & - i f_{(x+1,y)\downarrow}^\dagger f_{(x,y)\downarrow} - i f_{(x,y+1)\downarrow}^\dagger f_{(x,y)\downarrow} + i f_{(x+1,y+1)\downarrow}^\dagger f_{(x,y)\downarrow}] \\
 & + t_1' [e^{i\pi/3} f_{(x+1,y)\uparrow}^\dagger f_{(x,y)\downarrow} - f_{(x,y+1)\uparrow}^\dagger f_{(x,y)\downarrow} + e^{2i\pi/3} f_{(x+1,y+1)\uparrow}^\dagger f_{(x,y)\downarrow} \\
 & + e^{2i\pi/3} f_{(x+1,y)\downarrow}^\dagger f_{(x,y)\uparrow} + f_{(x,y+1)\downarrow}^\dagger f_{(x,y)\uparrow} + e^{i\pi/3} f_{(x+1,y+1)\downarrow}^\dagger f_{(x,y)\uparrow}] \\
 & + t_2' [e^{i\pi/6} f_{(x+1,y-1)\uparrow}^\dagger f_{(x,y)\downarrow} + e^{5i\pi/6} f_{(x+1,y+2)\uparrow}^\dagger f_{(x,y)\downarrow} - i f_{(x-2,y-1)\uparrow}^\dagger f_{(x,y)\downarrow} \\
 & + e^{5i\pi/6} f_{(x+1,y-1)\downarrow}^\dagger f_{(x,y)\uparrow} - i f_{(x-2,y-1)\downarrow}^\dagger f_{(x,y)\uparrow} + e^{i\pi/6} f_{(x+1,y+2)\downarrow}^\dagger f_{(x,y)\uparrow}] + h.c. \}
 \end{aligned}$$

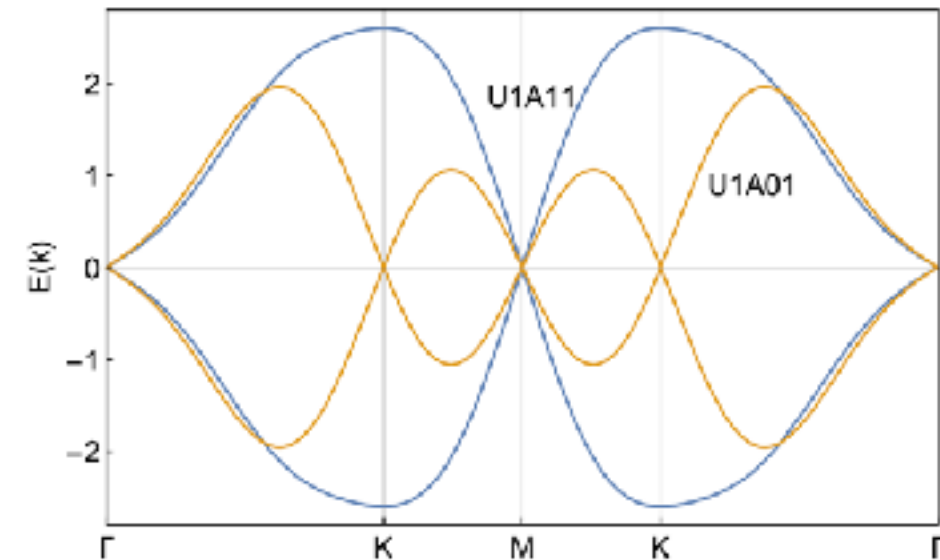
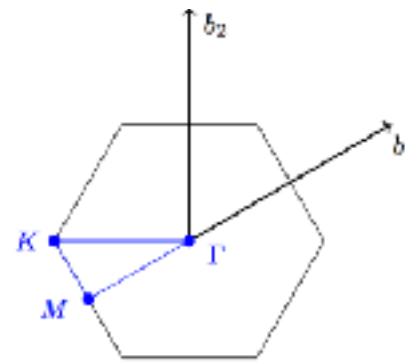


FIG. S-5. Spinon dispersions $E_{1,2}(\mathbf{k})$ along the line Γ -K-M-K- Γ for U1A11 and U1A01 states.

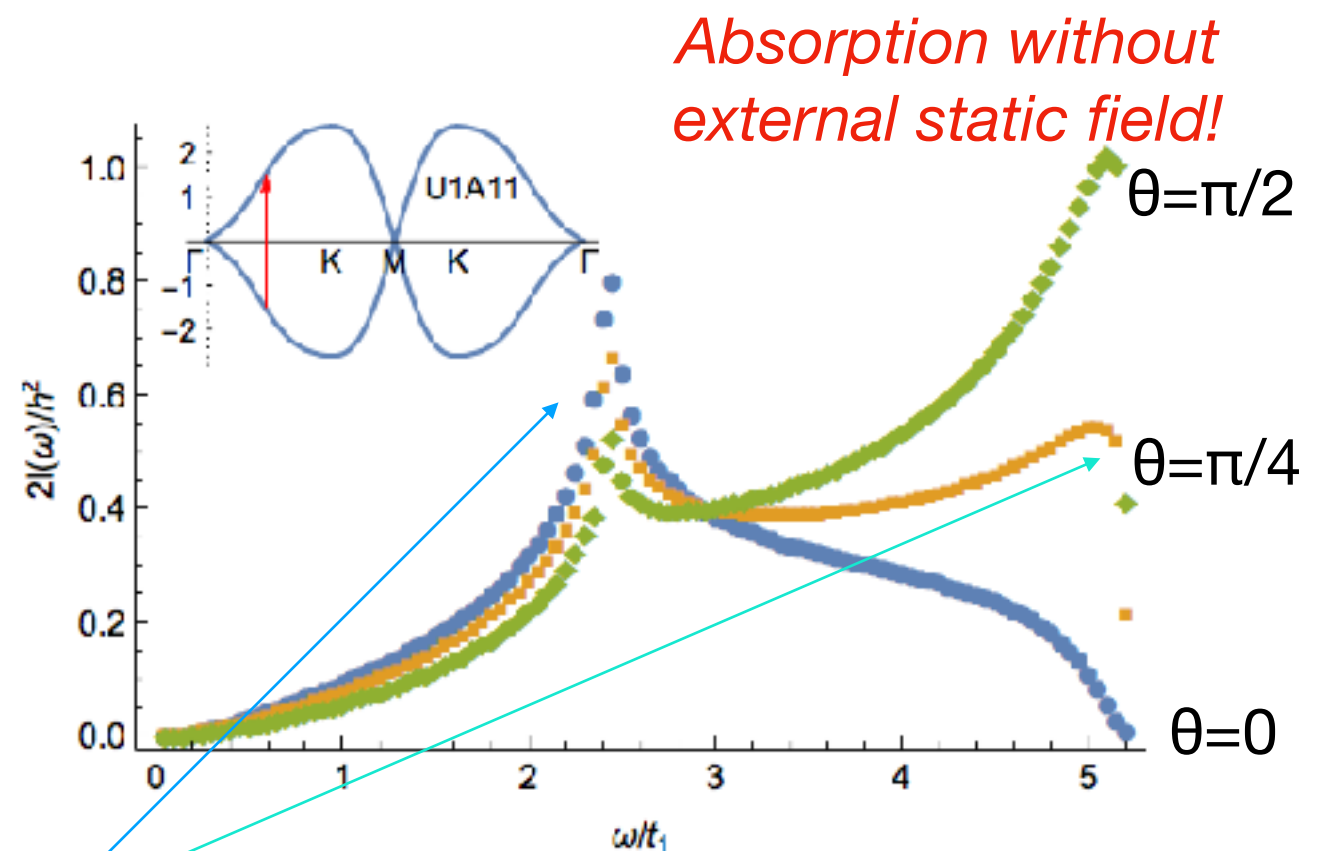
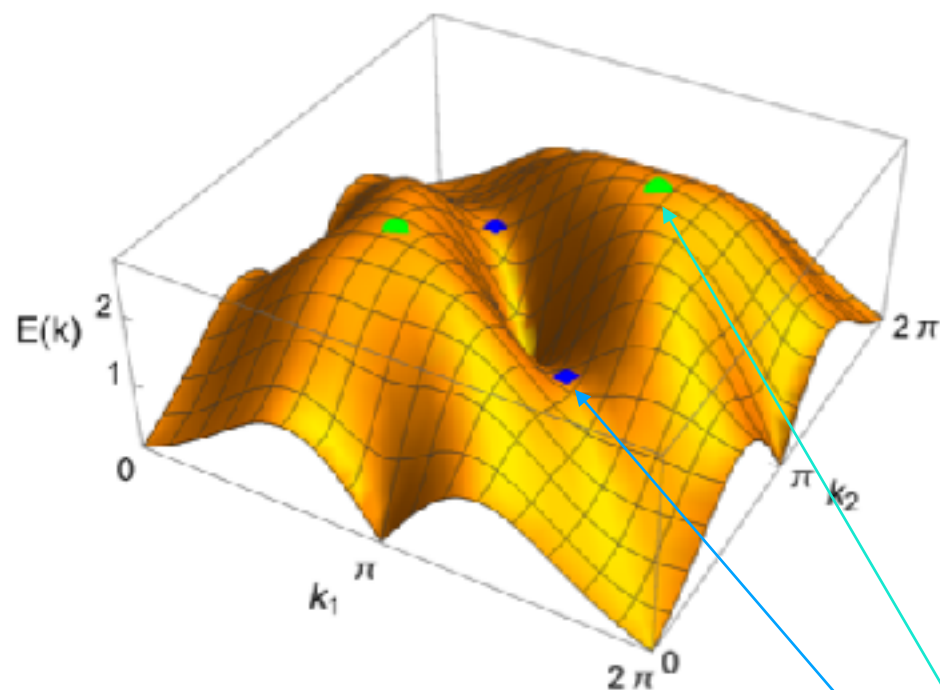
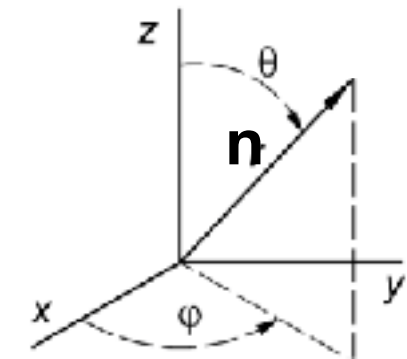
Spinon magnetic resonance

AC magnetic field couples to the total spin $S_{\mathbf{r}}^a = \frac{1}{2} f_{\mathbf{r}\alpha}^\dagger \sigma_{\alpha\beta}^a f_{\mathbf{r}\beta}$

$$V(t) = \hbar e^{-i\omega t} \mathbf{n} \cdot \frac{1}{2} \sum_{\mathbf{r}} (f_{\mathbf{r}\uparrow}^\dagger, f_{\mathbf{r}\downarrow}^\dagger) \boldsymbol{\sigma} \begin{pmatrix} f_{\mathbf{r}\uparrow} \\ f_{\mathbf{r}\downarrow} \end{pmatrix}$$

Rate of energy absorption $I(\omega) = -\omega \chi''_{nn}(\omega) |h|^2 / 2$

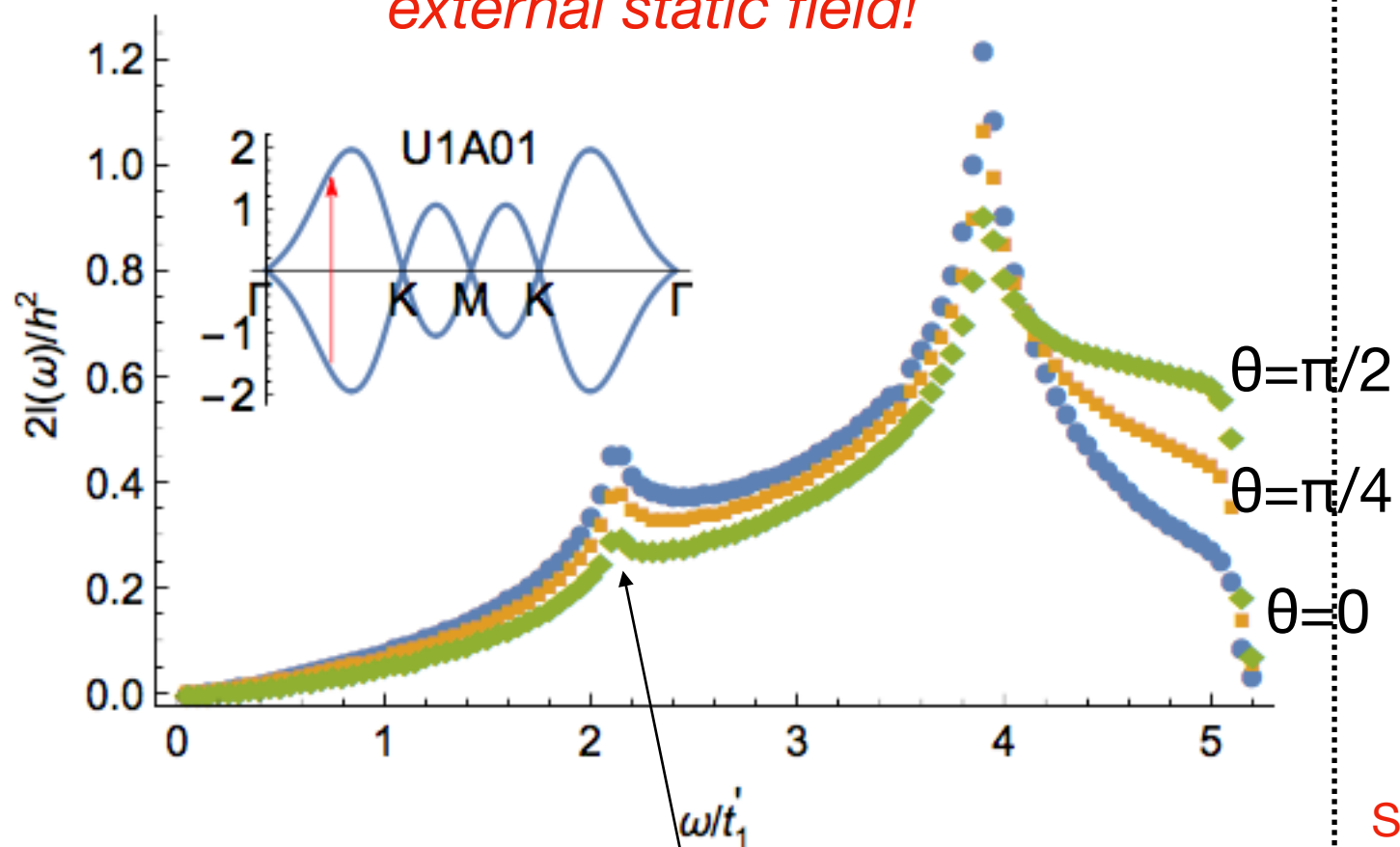
Dynamic susceptibility at $\mathbf{q}=0$
$$\chi_{nn}(\omega) = \frac{1}{4N} \sum_{\mathbf{k}} \frac{n_{\mathbf{k}\alpha} - n_{\mathbf{k}\beta}}{\omega + E_{\alpha}(\mathbf{k}) - E_{\beta}(\mathbf{k}) + i0} \times (U_{\mathbf{k}}^\dagger \sigma^a U_{\mathbf{k}})_{\alpha\beta} (U_{\mathbf{k}}^\dagger \sigma^b U_{\mathbf{k}})_{\beta\alpha} \hat{n}^a \hat{n}^b$$



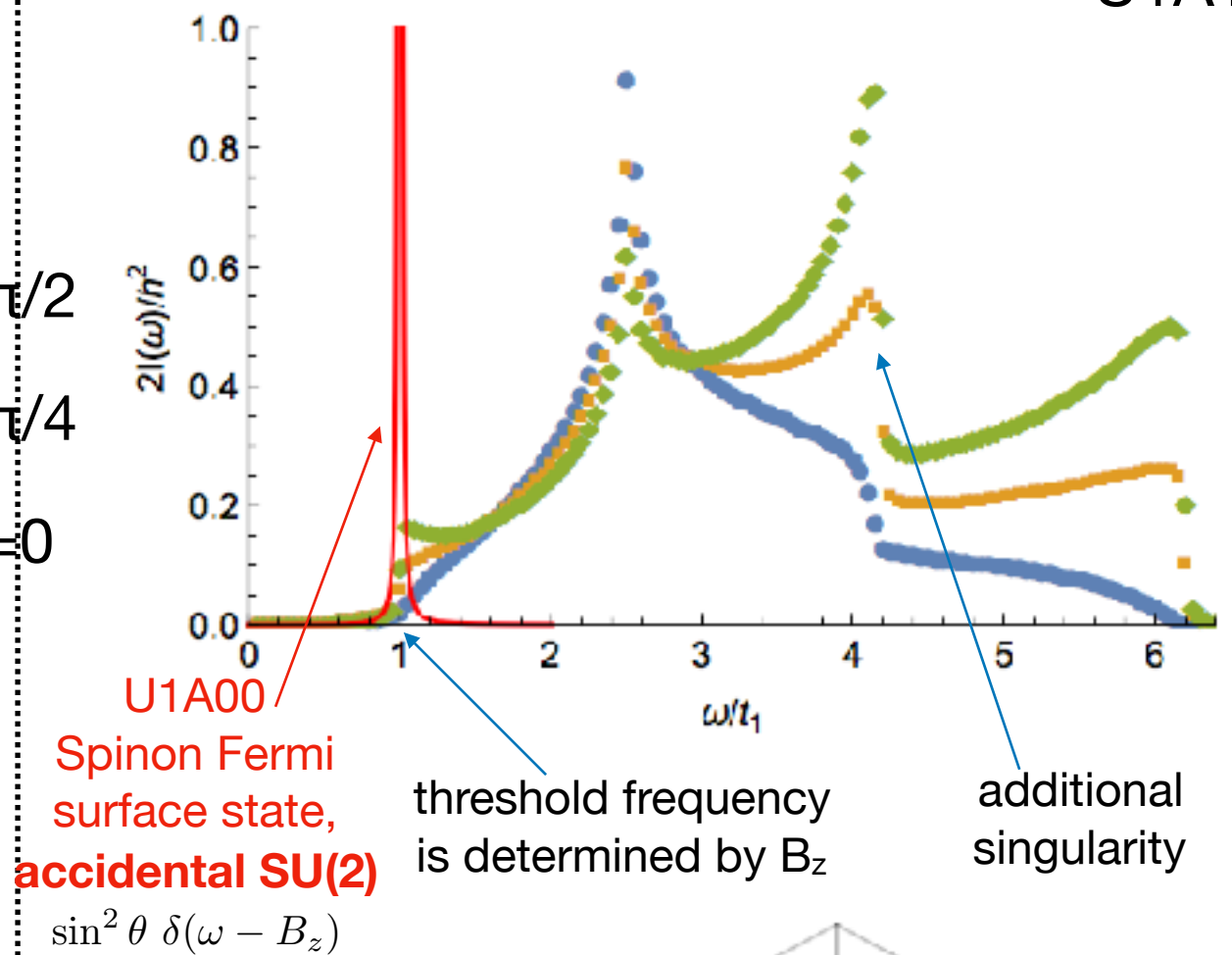
van Hove singularities

Absorption without external static field!

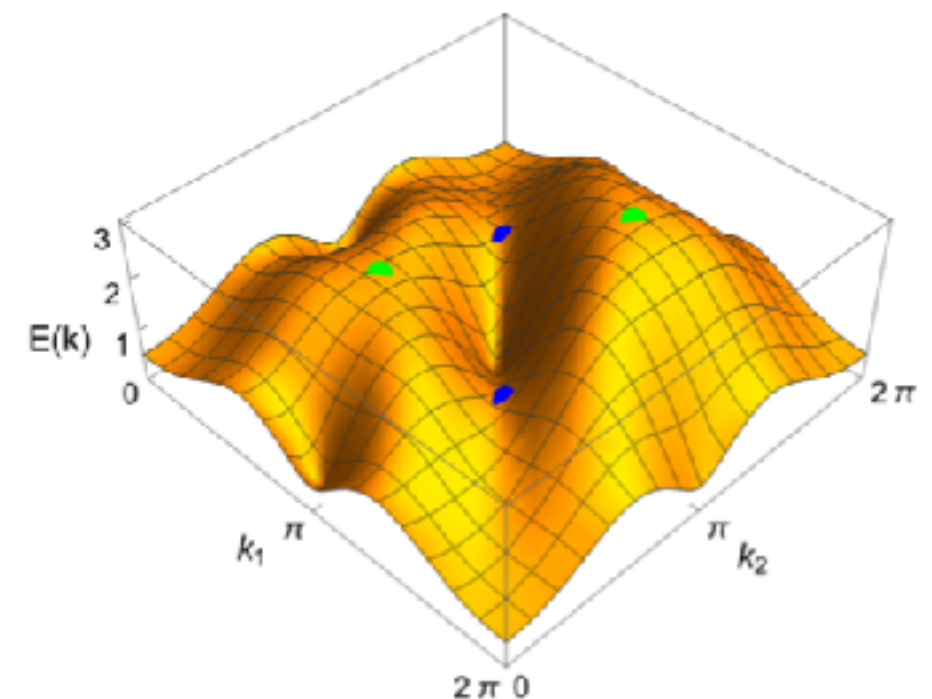
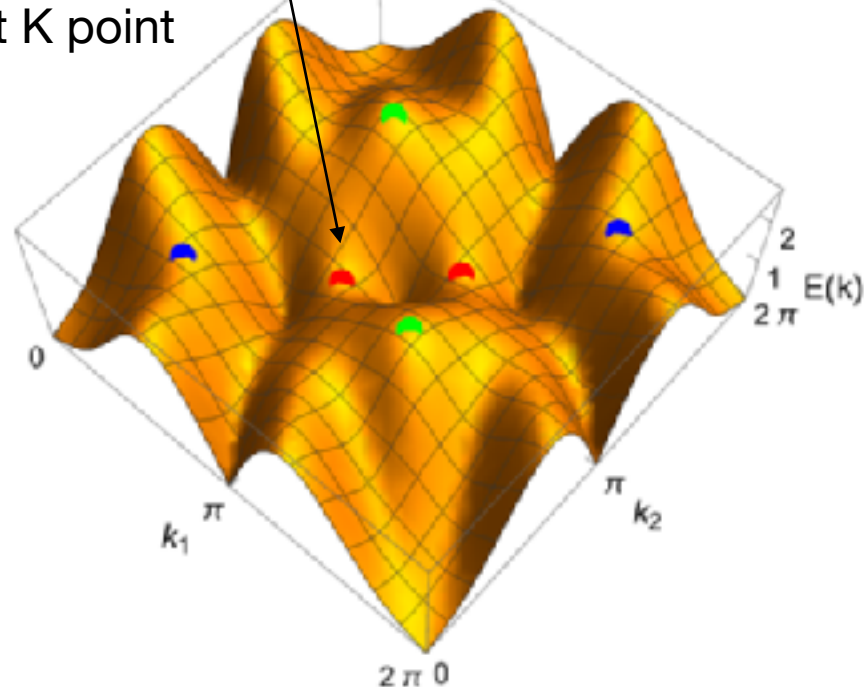
Absorption without external static field!



With magnetic field along Z
U1A11



Additional extremum in the spinon spectrum due to symmetry-enforced Dirac touching at K point



Existing ESR in YbMgGaO4

Y. Li, G. Chen, W. Tong et al,
Phys. Rev. Lett. 115, 167203 (2015).

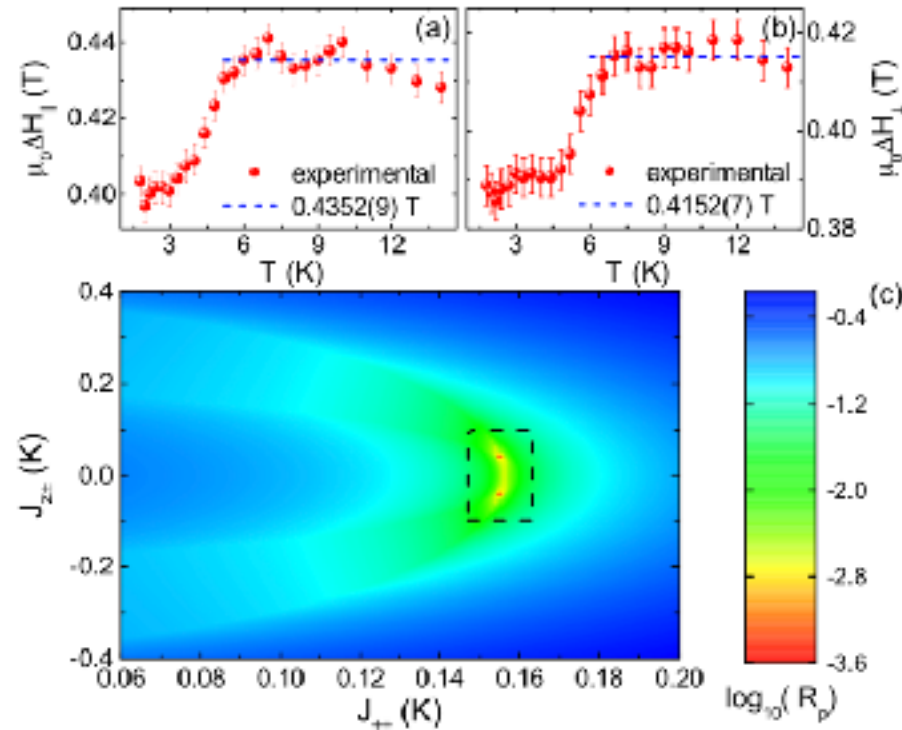


FIG. 3. (Color online.) The temperature dependence of ESR linewidths (a) parallel and (b) perpendicular to the c axis. The dashed lines are the corresponding constant fits to the ESR linewidth data at $T > 6$ K. (c) The deviation, R_p , of the experimental ESR linewidths from the theoretical ones for YbMgGaO₄. The dashed rectangle gives the optimal parameters $|J_{\pm\pm}| = 0.155(9)$ K and $|J_{z\pm}| = 0.04(10)$ K.

Minimum temperature: 1.8 K

X. Zhang, F. Mahmood, M. Daum et al,
arXiv: 1708.07503.

Model	A	B	B*	C
J_1^{zz} (meV)	0.126	0.164	0.151(5)	0.149(5)
$J_1^{\pm}$ (meV)	0.109	0.108	0.088(3)	0.085(3)
$J_1^{\pm\pm}$ (meV)	0.013	0.056	0.13(2)	0.07(6)
$ J_1^{z\pm} $ (meV)	0	0.098	0.1(1)	0.1(1)
J_2/J_1	0.22	0	0	0.18(7)
$g_{ }$	3.72	3.72	3.81(4)	3.81(4)
$g_{\perp}$	3.06	3.06	3.53(5)	3.53(5)

TABLE I. Exchange parameters for different models derived from fitting the spin-wave dispersions. Models A and B are from [60] and [43], respectively. Model C is from our global fit to the TDTS and INS data. Model B* is from a global fit to the data by ignoring NNN interactions, i.e. $J_2 = 0$. Uncertainties in the values represent the 99.7% confidence interval (3 s.d.) in extracting the fitting parameters.

Lower the temperature to
see the spinon effect!
 $T \sim 0.1$ K

Linewidth *vs* line shape

- Spinon band structure determines *line shape* of absorption (discussed previously).
- Interactions determine $\hbar, T$ -dependent *line width* !

Ideal U(1)
spin liquid

$$L_{u(1)} = \psi_{\alpha}^{\dagger} \left(\partial_t - i\mathbf{A}_0 + \epsilon(\nabla - i\vec{\mathbf{A}}) \right) \psi_{\alpha}$$



Rashba-like
perturbation
due to spin orbit
interaction

$$\delta L_R = \alpha_R \psi_{\alpha}^{\dagger} \left((p_x + \mathbf{A}_x) \sigma^y - (p_y + \mathbf{A}_y) \sigma^x \right) \psi_{\alpha}$$

Mori-Kawasaki formalism

Retarded spin GF $G_{S^+ S^-}^R(\omega) \propto 1/(\omega - h - \Sigma(\omega))$

Line width $\eta(\omega = h) = \text{Im}\Sigma(\omega = h) = -\frac{\text{Im}\{G_{A A^\dagger}^R(\omega)\}}{2\langle S^z \rangle}$

Perturbation is encoded in the **composite** operator
(depends on polarization of microwave radiation!)

$$\mathcal{A} = [\delta H_R, S^+] = -2i\alpha_R \sum_{p,q} \psi_{p+q}^\dagger \sigma^z \psi_p (A_{x,q} - iA_{y,q})$$

$$\eta(h) \sim \alpha_R^2 \int d\epsilon [1 + n_B(\epsilon) + n_B(h - \epsilon)] \text{Im}G_{S_q^z S_{-q}^z}^R(\epsilon) \text{Im}G_{A_q^- A_q^+}^R(h - \epsilon)$$

Gauge field propagator $\text{Im}G_{A_q^- A_q^+}^R(\nu) = \frac{\gamma q \nu}{\gamma^2 \nu^2 + \chi^2 q^6}$

`Particle-hole' spinon continuum $\text{Im}G_{S_q^z S_{-q}^z}^R(\epsilon) = \frac{m}{2\pi} \frac{\epsilon}{\sqrt{v^2 q^2 - \epsilon^2}} \Theta(vq - |\epsilon|)$



Preliminary results for perturbed U(1) spin liquid

OS, Balents, in progress...

$T = 0,$
 $h \gg T$

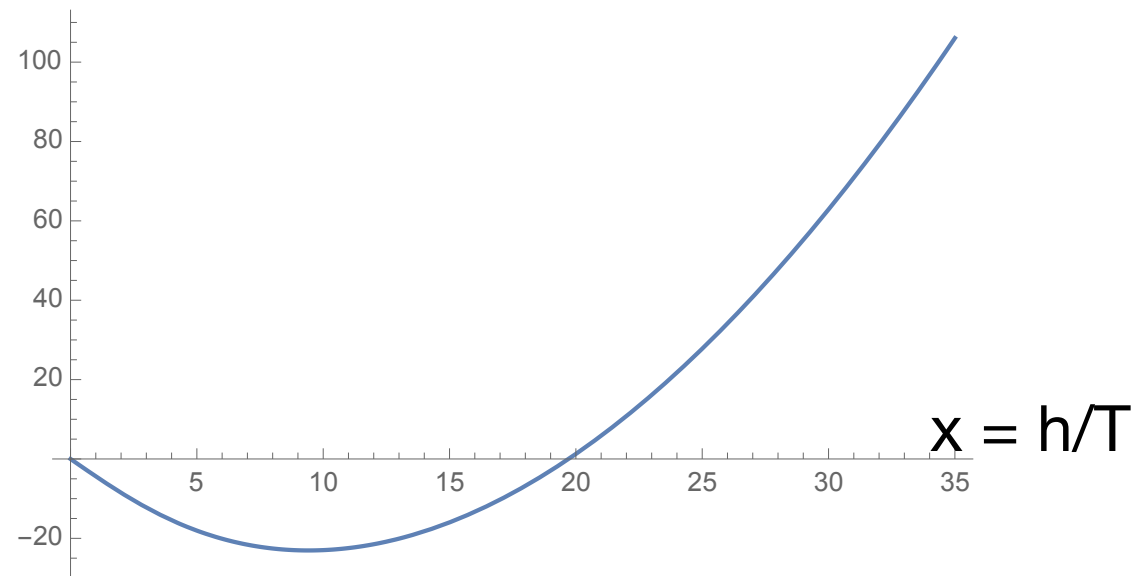
$$\eta \sim \alpha_R^2 \omega^{5/3} / h \sim h^{2/3}, h > 0$$

$$\eta \sim \omega^{2/3}, h = 0$$

$T > 0,$
 $h \ll T$

$$\eta = \frac{1}{2\chi_u h} \left(\frac{mT}{8\pi\chi} + \tilde{c}_0 T^{5/3} f\left(\frac{h}{T}\right) \right) \sim \frac{T}{h} + T^{2/3}$$

Scaling
function
 $f(x)$



$$f(x) \rightarrow -4.4x \text{ for } x \ll 1; f(x) \rightarrow 0.75x^{5/3} \text{ for } x \gg 1$$

Conclusion:

Spinon magnetic resonance is generic feature of spin liquids with significant **spin-orbit interaction** and fractionalized excitations

Main features:

- **broad continuum response**
- **zero-field absorption** (polarized terahertz spectroscopy)
- **strong polarization dependence**
- **van Hove singularities of spinon spectrum**
- **interesting and varying h, T dependence of the *resonance line width***

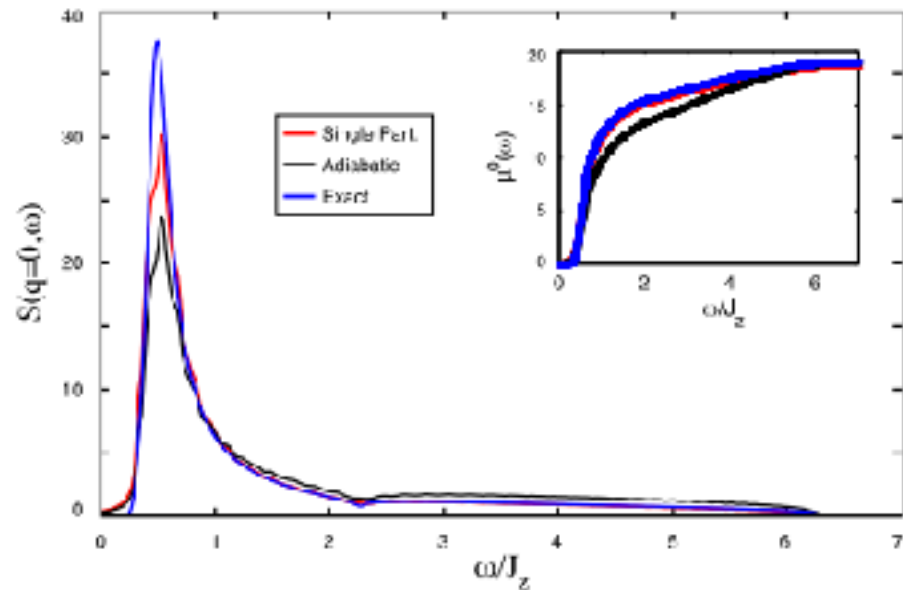
Kitaev model and α -RuCl₃

PHYSICAL REVIEW B **92**, 115127 (2015)



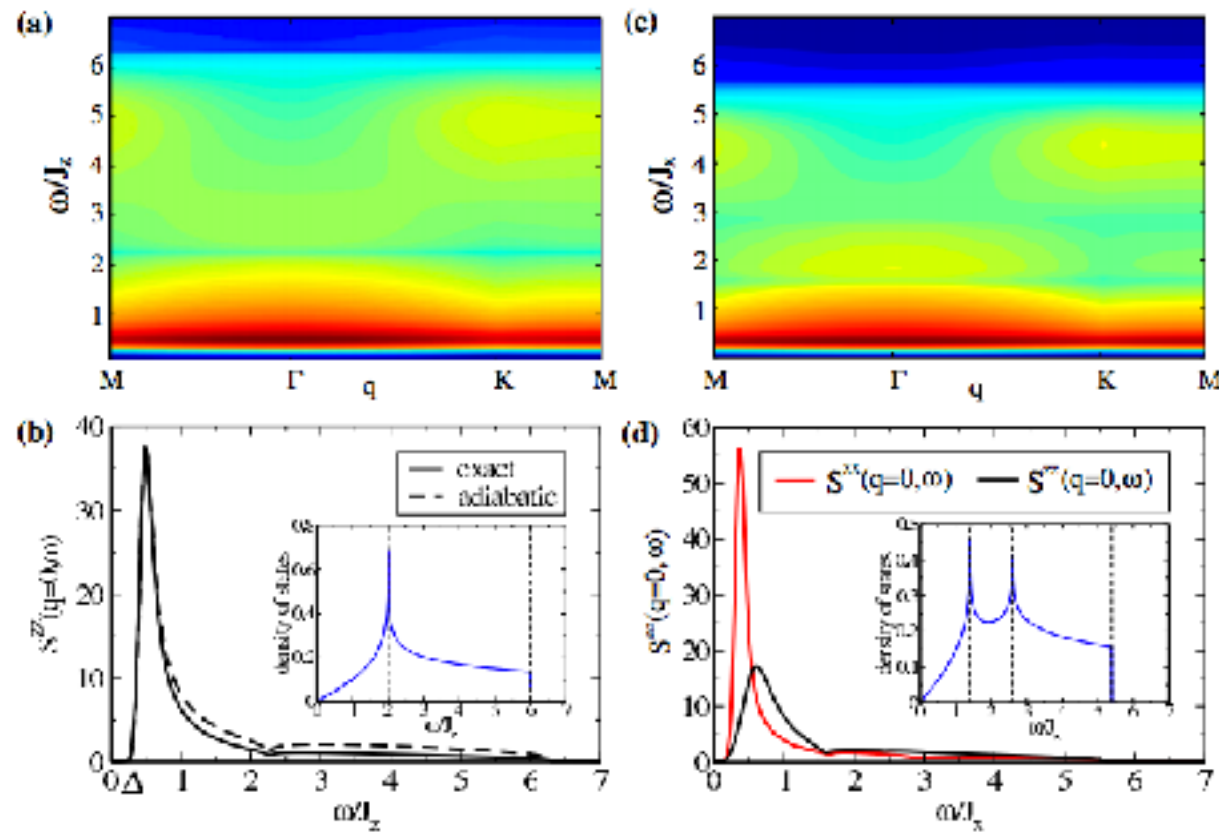
Dynamics of fractionalization in quantum spin liquids

J. Knolle,^{1,2,*} D. L. Kovrizhin,^{1,3} J. T. Chalker,⁴ and R. Moessner²



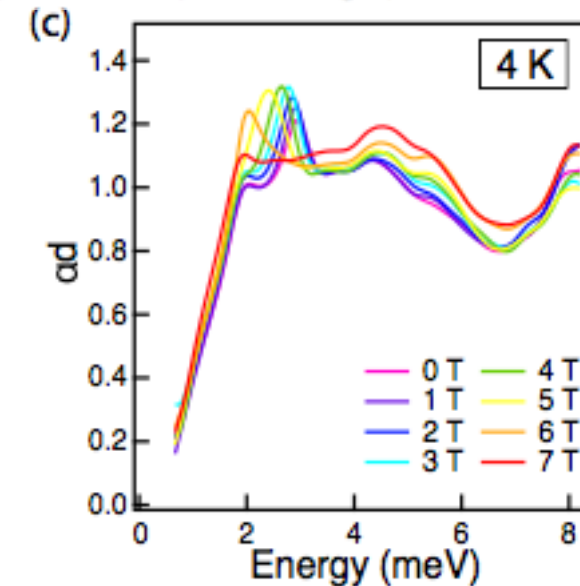
PRL **112**, 207203 (2014)

PHYSICAL REVIEW LETTERS



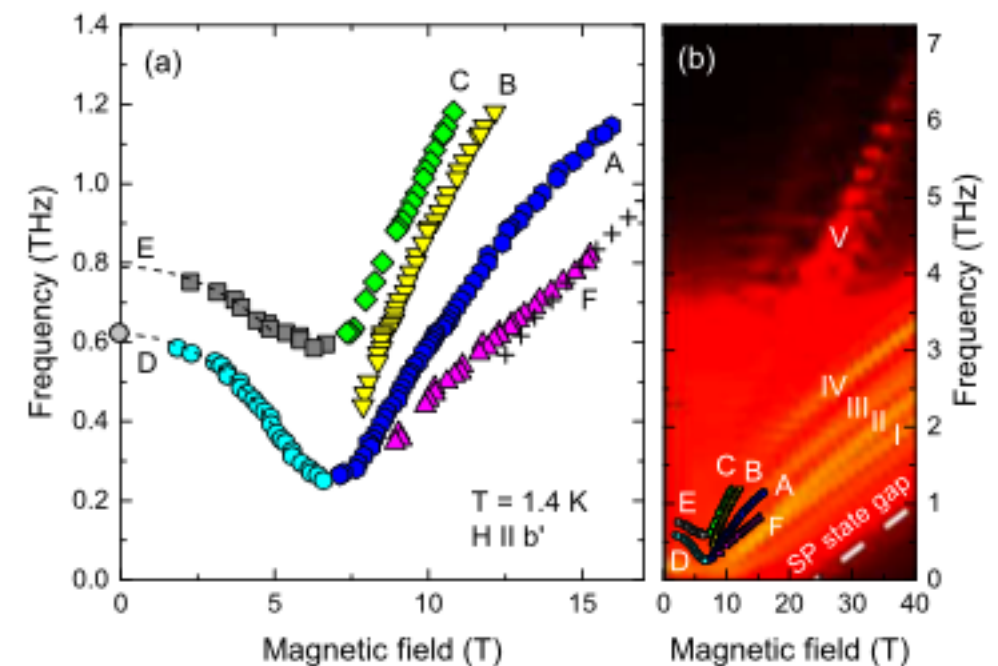
Antiferromagnetic resonance and terahertz continuum in α -RuCl₃

A. Little,^{1,2} Liang Wu,^{1,2,3,*} P. Lampen-Kelley,^{4,5} A. Danerjee,⁶ S. Patankar,^{1,2} D. Rees,^{1,2} C. A. Bridges,⁷ J.-Q. Yan,⁸ D. Mandrus,^{4,5} S. E. Nagler,^{6,9} and J. Orenstein^{1,2}



Direct observation of the field-induced gap in the honeycomb-lattice material α -RuCl₃

A. N. Ponomarev,¹ E. Schulze,^{1,2} J. Wosnitza,^{1,2} P. Lampen-Kelley,^{3,4} A. Danerjee,⁵ J. Q. Yan,³ C. A. Bridges,⁶ D. G. Mandrus,³ S. E. Nagler,⁵ A. K. Kolezhuk,^{7,8} and S. A. Zvyagin¹



Field evolution of magnons in α -RuCl₃ by high-resolution polarized terahertz spectroscopy

Liang Wu,^{1,2,3,*} A. Little,^{1,2} E. E. Aldape,¹ D. Rees,^{1,2} E. Thewalt,^{1,2} P. Lampen-Kelley,^{4,5} A. Banerjee,⁶ C. A. Bridges,⁷ J.-Q. Yan,⁸ D. Boone,^{9,10} S. Patankar,^{1,2} D. Goldhaber-Gordon,^{11,10} D. Mandrus,^{4,5} S. E. Nagler,⁶ E. Altman,¹ and J. Orenstein^{1,2,†}

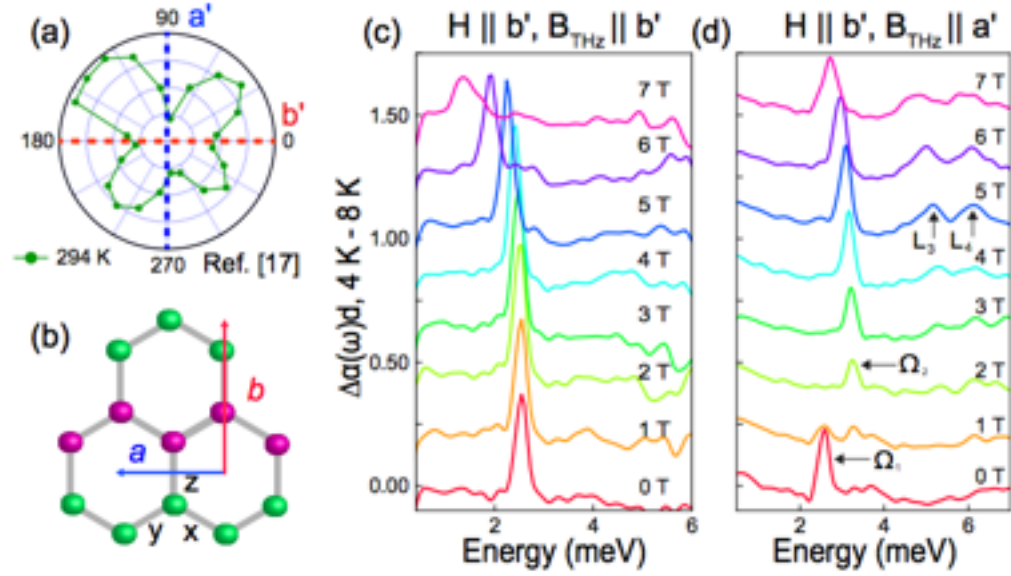


FIG. 1: (a) Transmitted THz electric field amplitude at $T = 294$ K as a function of sample angle. Blue and red lines represent the minimum transmission axes at a' and b' (b) Schematic of honeycomb structure showing a and b monoclinic axes relative to Ru-Ru bonds. Color of atoms illustrates zigzag order. Bond labels x , y , and z denote the component of the spin interacting along a given bond in the Kitaev model. (c) Magnon absorption as a function of frequency for $H \parallel b' \parallel B_{THz}$ and $H \parallel b' \perp B_{THz}$ respectively. The magnon contribution is extracted from the total THz absorption by subtracting a reference at $T = 8$ K, above T_N , from a $T = 4$ K spectrum at each field. Traces are offset for clarity.

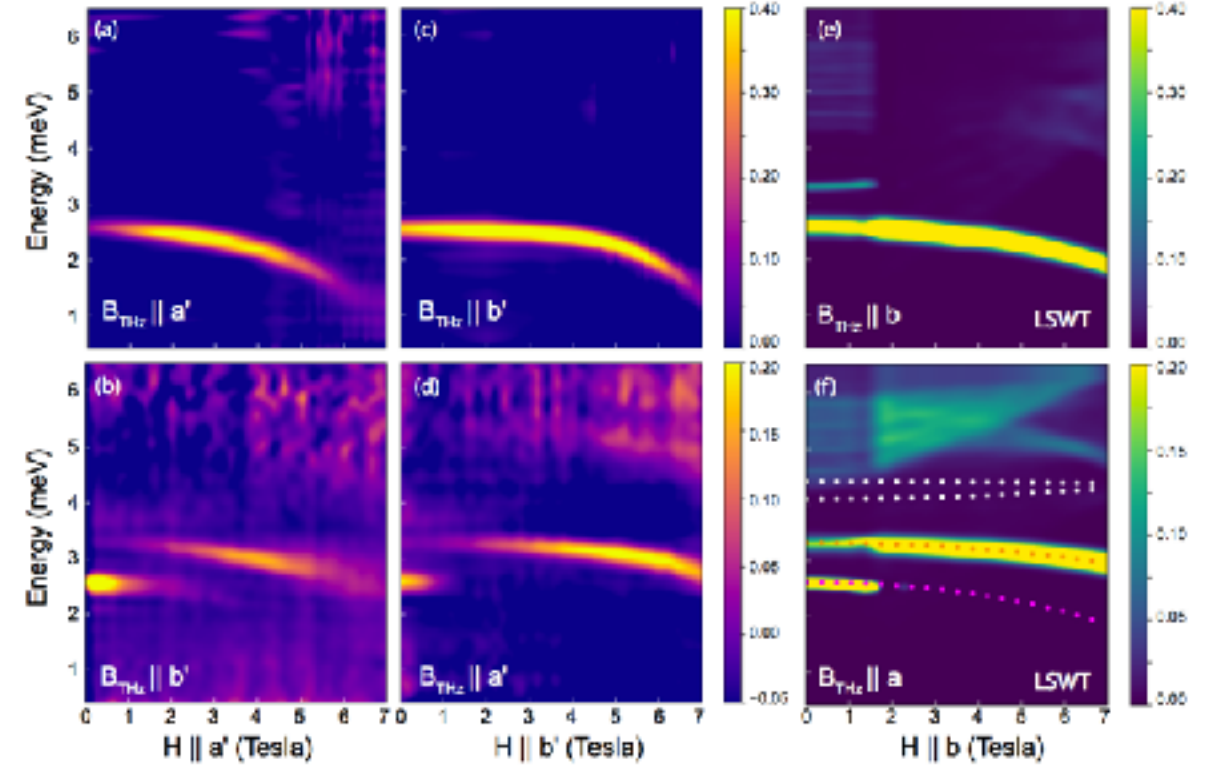


FIG. 2: Magnon energies and absorption strengths at $Q = 0$ as a function of external in-plane magnetic field, H . Experimental

Topical Review

Electrodynamics of quantum spin liquids

Martin Dressel¹ and Andrej Pustogow¹

¹Physikalisches Institut, Universität Stuttgart, Pfaffenwaldring 57, 70550 Stuttgart, Germany

Examples of
current literature

$$200 \text{ cm}^{-1} = 6 \text{ THz}$$

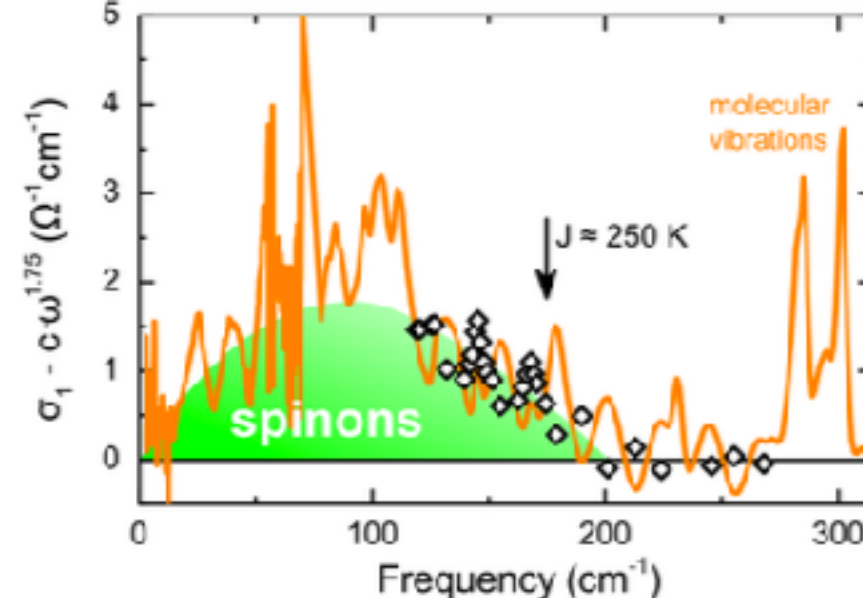


Figure 13. After subtracting the power-law background related with the Mott–Hubbard band, we obtain a broad low-energy feature below approximately 200 cm^{-1} , which is close to the antiferromagnetic exchange energy of β' - $\text{EtMe}_3\text{Sb}[\text{Pd}(\text{dmit})_2]_2$. Hence, we associate it with the coherent spinon Fermi surface predicted previously [60, 62]. The dome-like shape of the spinon contribution arises due to the rapid decay towards $\omega \rightarrow 0$, consistent with a power law such as the postulated ω^{-2} behavior [62]. This feature becomes visible when the background conductivity is sufficiently suppressed due to opening of the Mott gap. The strong, narrow modes at higher frequencies correspond to vibrational features.

Antiferromagnetic Resonance and Terahertz Continuum in α - RuCl_3

A. Little,^{1,2} Liang Wu,^{1,2,3,*} P. Lampen-Kelley,^{4,5} A. Banerjee,⁶ S. Putankar,^{1,2} D. Rees,^{1,2} C. A. Bridges,⁷ J.-Q. Yan,⁸ D. Mandrus,^{4,5} S. E. Nagler,^{6,9} and J. Orenstein^{1,2}

Spinons? ←

$$4 \text{ meV} = 1 \text{ THz}$$

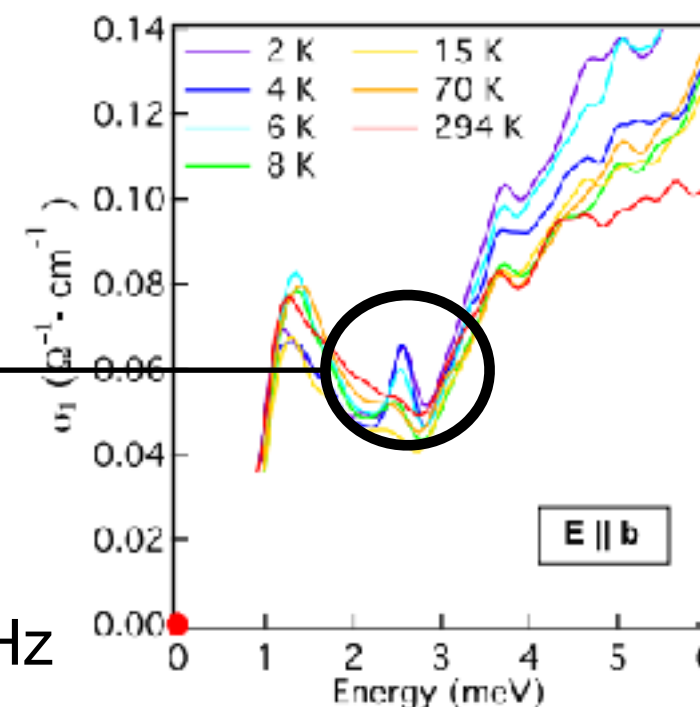


FIG. 3. Absorption spectra interpreted as optical conductivity, with $\mathbf{E}$ parallel to $\mathbf{b}$.